Monetary Union Begets Fiscal Union

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Abstract

We propose a mechanism through which monetary union between countries leads to a stronger fiscal union. Although fiscal risk-sharing is valuable under any monetary regime, given nominal rigidities it is more important within a monetary union, when exchange rates can no longer adjust to offset shocks. As a result, countries in a monetary union are capable of achieving better risk-sharing, partly overcoming their lack of commitment. Still, equilibria without fiscal cooperation remain possible and imply inefficient cross-country dispersion in output. A proactive central bank can encourage transfers by providing extra accommodation when fiscal union is under stress.

Transfer criterion
Countries that agree to compensate each other for adverse shocks form an optimum currency area.

Baldwin and Wyplosz (2012), Chapter 15

1 Introduction

A simplified narrative of the recent crisis in the Eurozone can be given as follows. Following the adoption of the single currency, a number of "periphery" countries progressively lost competitiveness as their real unit labor costs grew faster than the union average. When the global financial crisis hit in 2008, the accumulated internal imbalances came into sharp focus. Given their fixed nominal exchange rate vis-à-vis other Eurozone members, the only way periphery countries could achieve the real depreciation needed to regain competitiveness was through a painful process of economic contraction bringing about falls in domestic prices. Indeed, this adjustment process was so damaging to periphery economies that there was mounting speculation that some of them might leave the Euro. Monetary union was under stress. But it ultimately did not break: Eurozone

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countries were strongly bound to their single currency despite the large stabilization costs that it induced.

Meanwhile, periphery countries' large external and fiscal deficits became increasingly difficult to finance. Some of the financing was bridged through bailout packages, but their size was limited by reluctance from core country taxpayers who were supposed to fund them. Eurozone fiscal union was only implicit, and core countries hit their participation constraint.

Our paper studies fiscal unions subject to such participation constraints, contrasting their role inside and outside a monetary union. To capture features of fiscal union that resemble those of the Eurozone today, we assume that countries have limited ability to commit to risk-sharing. Specifically, we require that cross-country transfers always be part of a subgame-perfect equilibrium of a repeated game: in our model, countries only make transfers if these transfers are backed by credible promises of future reciprocity. This emphasis on reciprocity is supported by a recent study from the IMF (Allard et al. (2013)), which finds that "with a risk-sharing mechanism in place over a sufficiently long period, all current euro area members would have benefited from transfers at some point in time". Meanwhile, we capture the costs of monetary union by introducing nominal rigidities. When countries are part of a monetary union, the central bank can stabilize the union as a whole, but not each individual country—leading to overheating in some countries and to recessions in others. These stabilization costs are absent under independent monetary policy, where each country's central bank can stabilize its own economy separately.

Our primary result is that monetary union enhances fiscal union. The stabilization costs induced by monetary union *also* make countries more willing to share risks. This is due to an interaction between the degree of risk-sharing and the ability of the union-wide central bank to stabilize its members: when countries share risks better, there is less divergence in the stance of monetary policy that is appropriate for each country individually. In our benchmark model, this idea is illustrated starkly by the *risk-sharing miracle*: when its member countries share risks perfectly, the union-wide central bank is able to stabilize all of them simultaneously.

Our paper proposes a mechanism through which monetary union between countries leads to a stronger fiscal union. By doing so, it contributes to *sequencing theory*, a field of international relations that studies how one type of economic cooperation can lay the foundation for the next. This literature often takes as a starting point an interpretation of Balassa (1962), according to which regional integration takes place by progressively climbing the steps of the "integration staircase" depicted in figure 1 (Gustavsson (1999), Estevadeordal and Suominen (2008), Baldwin (2012)). Central to sequencing theory is the existence of spillovers that make integration gather momentum by begetting further cooperation in other areas. For example, Haas (1958), in his famous study of the European Coal and Steal Community in the early 1950s, emphasizes the ability of the newly-founded institution to support special interest groups that pushed for broader economic integration, eventually leading to the European Economic Community. Our paper microfounds the spillovers that enable countries to climb the last step of the integration staircase: in a monetary union, the absence of fiscal union becomes more costly, and countries internalize this when deciding on fiscal cooperation.

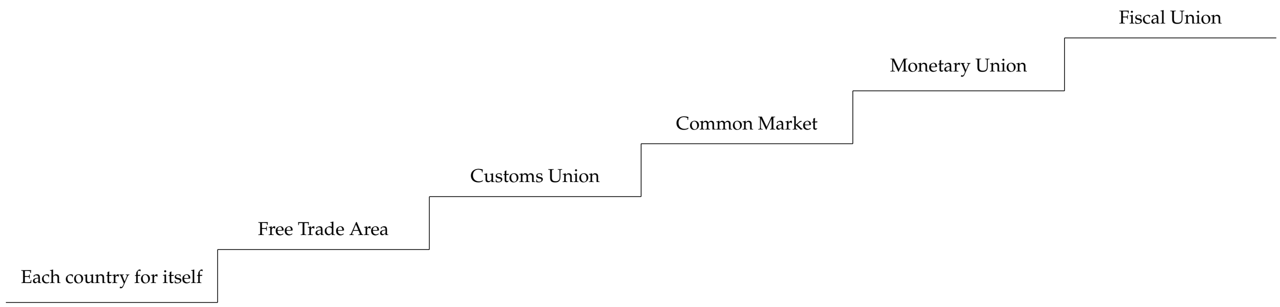

Figure 1: The integration staircase

Our paper also adds to the theory of Optimum Currency Areas (Mundell (1961), McKinnon (1963), Kenen (1969)) by proposing a new tradeoff, between stabilization costs and *risk-sharing benefits* of monetary union. While the cost side of the ledger—the difficulty of monetary union to cope with asymmetric shocks—has been well understood since at least Friedman (1953), the benefit side has lacked comparatively compelling microfoundations. The literature emphasizes such diverse advantages as the elimination of transactions costs or the gains from reduced uncertainty, which we find less tangible than the risk-sharing benefits stressed in this paper. Another recent paper revisiting the benefits side of the monetary union ledger is Chari, Dovis and Kehoe (2013). Their emphasis is on the benefits to central bank credibility, pointing out that a union may reduce overall inflationary bias to the extent that the shocks that create a desire to inflate are not perfectly correlated across countries. This channel is absent in our model, in which we shut off all forms of inflationary bias.

One way to read our paper is thus, in the light of the OCA literature, as a direct argument for why countries might form monetary unions despite their apparent costs. Our argument is that these costs may in fact be the seed of the benefits: once the monetary union is joined, the possibility of high stabilization costs in the absence of fiscal cooperation enforces the cooperation itself, and limits the incurrence of the costs.

Having established our main result, we go further and ask what the union-wide central bank can do, given additional commitment power, to proactively encourage the fiscal union. We find that it can help, by departing from its traditional role of aggregate stabilization and committing to accomodative monetary policy in volatile times. Accomodative monetary policy helps because it creates an overall boom in the union, which in turn effectively relaxes countries' participation constraints and facilitates transfers. This incentive effect is new to the literature on optimal monetary policy in currency unions.

Our model puts together two distinct strands of the literature. The first is the literature on limited commitment (Kehoe and Levine (1993), Coate and Ravallion (1993), Kocherlakota (1996), Alvarez and Jermann (2000), Ligon, Thomas and Worrall (2002)). This literature derives endogenous constraints on risk-sharing by giving agents the option to leave transfer arrangements at any point in time, and it focuses on the best outcomes that are sustainable given these constraints. When countries run an independent monetary policy, our setup reduces to the one described by this literature, and the same forces—the degree of patience, risk-aversion, and the persistence of the idiosyncratic endowment processes—drive the feasible amount of risk-sharing.

We combine the constraints on risk-sharing featured by the limited commitment literature with the constraints on monetary policy emphasized by the modern international economics literature on currency unions in the presence of nominal rigidities (Benigno (2004), Galí and Monacelli (2005), Galí and Monacelli (2008)). This literature provides a microfoundation for the stabilization costs that arise in currency unions, as the central bank is generally unable to fully stabilize each member country and must balance out the cross-country distortions it creates by setting its policy instrument at an intermediate level.

Our paper is close in spirit and in form to Farhi and Werning (2013), who also study the benefits of a fiscal union of the kind we describe — cross-country insurance arrangements — within a monetary union. While they focus on the constrained inefficiency of private arrangements and the need for government interventions to reach a constrained-optimal outcome, we shut off private international financial markets and study a constraint faced by the *governments* in their implementation of the optimal outcome. In doing so, we extend their framework to allow for a full game-theoretic analysis of monetary and fiscal policy. In most of our paper, the risk-sharing miracle holds and the constrained-optimal outcome is also first-best, a case which is of limited interest in Farhi and Werning (2013), but which we regard as capturing in an elegant way the widespread view that alignment of fiscal policy limits the costs of monetary union. Under the risk-sharing miracle, nominal rigidities only create a cost to the extent that a limited commitment friction binds and prevents countries from reaching a full risk-sharing outcome. As we briefly discuss, this would no longer be true if we allowed for shocks to preferences or nontradables productivity.

Another paper that discusses monetary union in the presence of a limited commitment friction is Fuchs and Lippi (2006). Their focus is on the short-term commitment benefits brought about by monetary union, in a situation where independent central banks might otherwise be tempted to follow beggar-thy-neighbor policies. They use a reduced form to specify country preferences over the level of the monetary policy instrument. In contrast, we abstract away from the interesting possibility that monetary union might break up, but we fully endogenize fiscal and monetary policy, assuming both policies maximize country welfare subject to the constraints of the environment (limited commitment and nominal rigidities). This allows us to study the rich ways in which these two frictions—and these two types of policies—interact.

Many recent policy discussions have been focused on the need to establish fiscal federalism in the Euro Area. Our model recognizes the importance of these efforts. Except in the special case where our mechanism is so powerful as to endogenously lead countries to share risks perfectly, the limited commitment friction does create costs, so it is valuable to try and mitigate it. In fact, the risk-sharing miracle implies that if countries could overcome the commitment friction altogether, they would be able to attain the first best. In practice, it has been difficult to get countries to sign formal agreements regarding contingent future fiscal transfers. We have two insights to add here. First, we emphasize that because it is in the private interest of countries to internalize the macroeconomic externalities associated with monetary union (Farhi and Werning (2013)), a stronger fiscal union might emerge on its own: the set of possible equilibria is enlarged, though countries may take time to move to the more cooperative one. Second, our normative analysis suggests that proactive monetary policy might be used as an imperfect substitute to fiscal union, nudging countries into sharing risks better.

The rest of our paper is organized as follows. Section 2 introduces our main framework, laying out our model's game-theoretic foundations and defining our equilibrium concept. It then proves a number of properties of equilibria which simplify the analysis in the rest of the paper. Among these are the risk-sharing miracle (Theorem 1) and the ability to sustain any subgame perfect equilibrium using strategies that revert to autarky following any deviation (Theorem 2). Section 3 considers a case where countries' endowments satisfy a simple symmetry condition, and delivers two main results that substantiate our claim that monetary union enhances fiscal union. Theorem 3 shows any risk-sharing arrangement that is sustainable under independent monetary policy is also sustainable under monetary union. It is a sharp illustration of the sense in which the monetary union improves risk-sharing. Theorem 4 shows that, under certain cases, the monetary-union-induced improvement in risk sharing is so powerful that it can take countries all the way from autarky to first best. Section 4 proposes a normative analysis of monetary policy when fiscal union is subject to a limited commitment friction. Theorem 5 shows that it is valuable to provide aggregate stimulus in times of high volatility in order to create a macroeconomic environment favorable to transfers between countries. Section 5 concludes. All proofs are in Appendix A.

2 Main framework

2.1 Fundamentals

Two countries $i = 1, 2$ live forever and have identical preferences. Each values the stream of tradables consumption $\left\{C^i_{T,t}\right\}$, nontradables consumption $\left\{C^i_{NT,t}\right\}$ and labor $\left\{N^i_t\right\}$ according to the utility function

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u\left(C^i_{T,t}, C^i_{NT,t}, N^i_t\right)\right]$$

where we specify the felicity function to be log in tradables and nontradables, and isoelastic in labor:

$$u\left(C_T, C_{NT}, N\right) = \log\left(C_T\right) + \alpha\left(\log\left(C_{NT}\right) - \frac{N^{1+\phi}}{1+\phi}\right) \tag{1}$$

All goods are perishable, tradables goods are nonproduced, nontradable goods have to be consumed where they are produced, and labor is immobile across countries.

There exists another country in the world that is also endowed with tradables. The only purpose of this country in the model is to provide the reference unit of account, as Section 2.1.3 will discuss.

2.1.1 Tradable goods

Aggregate uncertainty is decribed by a finite-state Markov process $\{s_t\}$ with elements $s_t \in \mathbf{S}$ and transition matrix Π. A history of length t is denoted by $s^t = (s_0, \ldots, s_t)$. We write $s^\tau \succeq s^t$ to indicate that s^τ is a successor node of s^t.

Each country has a risky endowment $E_T^i(s_t)$ of an identical, freely tradable good. The aggregate state s_t thus determines the distribution of tradable endowments across the two countries. For now, tradable endowment shocks are the only source of uncertainty in the model.

Assumption 1 (External balance)**.** *The union achieves external balance in each history s^t:*

$$C_T^1\left(s^t\right) + C_T^2\left(s^t\right) = E_T^1\left(s_t\right) + E_T^2\left(s_t\right) \equiv E_T\left(s_t\right) \quad \forall s^t = (s_0, \ldots, s_t)$$

Assumption 2 (Strict benefits from tradables risk-sharing)**.** *For all $s \in \mathbf{S}$, there exists $s' \in \mathbf{S}$ such that*

$$\frac{E_T^1(s)}{E_T^2(s)} \neq \frac{E_T^1(s')}{E_T^2(s')}$$

Since countries have concave expected utility over tradable consumption, assumptions 1 and 2 together imply that there exist ex-ante utility gains from sharing risks by arranging state-contingent transfers of tradables. Like Farhi and Werning (2013), we call such an arrangement a "fiscal union". In our model, the extent of risk-sharing is limited by a commitment friction which we will soon describe.

2.1.2 Nontradable goods

Nontradable goods are produced from labor by a continuum of firms. We abstract from uncertainty regarding nontradable production, and discuss the consequences of relaxing this assumption in Section 3.3. In each country i, there is a continuum of firms $j \in [0,1]$ that each operate the simple technology

$$Y_{NT}^{i,j} = N^{i,j}$$

in each period[1]. The consumer's utility value from the consumption of each variety is given by the CES aggregator

$$C_{NT}^i = \left(\int_{j=0}^1 \left(C_{NT}^{i,j}\right)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \quad \epsilon > 1 \tag{2}$$

[1] To reduce notation, we suppress dependence on the time and state whenever this is unambiguous.

Consumption of each variety must equal production, $C_{NT}^{i,j} = Y_{NT}^{i,j}$, and labor market clearing requires $N^i = \int_j N^{i,j} dj$.

Our assumed preferences and production structure are intended to make the first-best level of nontradables a simple reference point. Since all firms have the same technology, efficiency requires them to produce equally, in which case $C_{NT}^{i,j} = N^{i,j} = C_{NT}^i = N^i$ for all j. Optimizing utility (1) subject to this constraint gives $C_{NT}^i = N^i = 1$.

Lemma 1. *An efficient allocation of production requires* $C_{NT}^{i,j} = N^{i,j} = 1, \forall i, j$

In equilibrium, production may depart from this efficient level as a result of monopoly power and nominal rigidities.

2.1.3 Rest of the world and units of account

In order to discuss exchange rate regimes, we need to be specific about units of account. We assume that the homogenous tradable good is traded as part of a world market, and that its foreign-currency price is normalized to $P_T^*\left(h^t\right) = 1$ in all histories h^t (h^t consists of the exogenous state s^t as well as the history of previous actions by all agents, as described below). The foreign currency, which we call the dollar, then provides a natural reference unit of account, and we assume that transfers between countries are specified in that unit of account. With this interpretation, assumption 1 amounts to imposing that the two countries have a closed capital account vis-à-vis the rest of the world.

We think of monetary policy as fixing the nominal exchange rate $\mathcal{E}^i\left(h^t\right)$ in each history—that is, the number of units of domestic currency it stands ready to buy or sell per dollar. By the law of one price for tradables, the exchange-rate policy of the central bank effectively fixes the domestic currency price of the tradable good at $P_T^i\left(h^t\right) = \mathcal{E}^i\left(h^t\right)$ in every history h^t. The key difference between flexible exchange rates and a monetary union is that, in the latter, the union-wide central bank has to set a common exchange rate $\mathcal{E}^1\left(h^t\right) = \mathcal{E}^2\left(h^t\right)$ in each history.

2.2 Timing and equilibrium

As ensured by Assumption 2, there are gains from risk-sharing in tradable goods. We study an environment where transfers between countries emerge as part of a subgame perfect equilibrium of a repeated game. Three distinct types of actors participate in this game: monopolistically competitive firms setting prices for nontradable goods, fiscal authorities for each country, and—depending on the monetary regime—either a common central bank for both countries or two independent central banks.

The timing within each period is given in Figure 2. We start by outlining the sequence of events informally, before describing each part in detail in Sections 2.2.1-2.2.4.

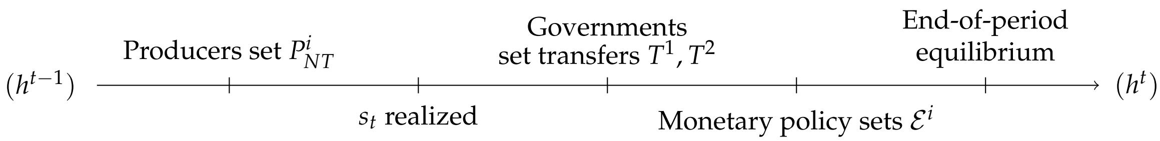

Figure 2: Timing

At the beginning of period t, h^{t-1} includes the history of all previous actions and states. Each actor in period t has a pure strategy conditional on both the incoming history h^{t-1} and any preceding actions within period t. First, before the state is realized, firms set nontradable goods prices (in the domestic unit of account) based on h^{t-1}, producing a price distribution φ^i_t in each country. Once the state s_t is realized, the fiscal authority in each country makes a transfer T^i based on h^{t-1}, $\{\varphi^i_t\}$, and s_t. As discussed in Section 2.1.3, this transfer is specified in the international unit of account. Next, exchange rates $\mathcal{E}^i_t$ are chosen either separately in each country or—in the case of a monetary union—commonly for both, based on h^{t-1}, $\{\varphi^i_t\}$, s_t, and $\{T^i_t\}$. Finally, based on the state and all actions thus far in the period, the end-of-period market determines production and consumption according to household demand.

In the following sections, we proceed by backward induction, describing how the outcome at each step within the period is determined, taking subsequent strategies as given. As depicted in Figure 2, there are four steps of interest: end-of-period equilibrium, monetary policy, fiscal policy, and pricesetting. These are the subjects of Sections 2.2.1-2.2.4, respectively.

2.2.1 End-of-period equilibrium

Households. At the end of the period, households in country i are faced with prices $P^{i,j}_{NT}$, P^i_T, and W^i, as well as profits $\psi^{i,j}$ earned from each firm j's production and a lump sum tax t^i from the domestic government. The nontradable goods prices $P^{i,j}_{NT}$ have already been set, while the prices P^i_T and W^i are determined on a Walrasian market. We assume that households do not have access to financial markets. We could allow them to access domestic financial markets without loss of generality: since the government has access to a lump-sum tax, Ricardian equivalence would hold.

Households optimally choose consumption

$$\left\{C^i_T, C^{i,j}_{NT}, N^i\right\} \in \arg\max_{\left\{\hat{C}^i_T, \hat{C}^{ij}_{NT}, \hat{N}^i\right\}} \left(\log\left(\hat{C}^i_T\right) + \alpha\left(\log\left(\hat{C}^i_{NT}\right) - \frac{\left(\hat{N}^i\right)^{1+\phi}}{1+\phi}\right)\right)$$
$$\text{s.t.} \quad P^i_T\hat{C}^i_T + \int_{j=0}^1 P^{i,j}_{NT}\hat{C}^{i,j}_{NT}\mathrm{d}j \leq P^i_T E^i_T + W^i\hat{N}^i + \int_{j=0}^1 \psi^{i,j}dj - t^i \qquad (3)$$

where $\hat{C}^i_{NT}$ is the aggregator in (2). The following lemma is derived from the first-order conditions of the problem.

Lemma 2. *At an optimum of the consumer problem, tradable and nontradable consumption are propor-*

tional:

$$C^i_{NT} = \alpha \frac{P^i_T}{P^i_{NT}} C^i_T \tag{4}$$

good-specific non-tradable demand is

$$C^{i,j}_{NT} = \left(\frac{P^{i,j}_{NT}}{P^i_{NT}} \right)^{-\epsilon} C^i_{NT} \tag{5}$$

and labor supply is

$$N^i = \left(\frac{W^i}{P^i_{NT}} \frac{1}{C^i_{NT}} \right)^{\frac{1}{\phi}} \tag{6}$$

where $P^i_{NT} = \left(\int_j \left(P^{i,j}_{NT} \right)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ *is the price index associated with* $\left\{ P^{i,j}_{NT} \right\}$.

Profits of nontradable goods producers. We assume that governments subsidize the labor cost of firms at a rate $\tau^i_L = -\frac{1}{\epsilon}$. Given its price $P^{i,j}_{NT}$, producer j honors demand $C^{i,j}_{NT}$ by hiring $N^{i,j}$ workers and generates profits

$$\psi^{i,j} = \left(P^{i,j}_{NT} - \left(1 + \tau^i_L \right) W^i \right) N^{i,j} \tag{7}$$

which are remitted to households as part of their budget (3).

Government. As determined earlier in the period, the government of country i sends an international transfer of T^i and receives one of T^{-i}, both denominated in dollars. It operates the labor tax, and rebates all profits to households. Its budget constraint, expressed in domestic currency units, is then

$$t^i = \mathcal{E}^i \left(T^{-i} - T^i \right) + \tau^i_L W^i \int_j N^{i,j} dj \tag{8}$$

Law of one price for tradable goods. As per its decision earlier in the period, the central bank sets its nominal exchange rate $\mathcal{E}^i$ against the dollar. Given this exchange rate, domestic residents can purchase tradable goods from the rest of the world at price $\mathcal{E}^i$ or from the domestic market at price P^i_T. Equilibrium requires that the two be equated, to prevent pure arbitrage profits:

$$P^i_T = \mathcal{E}^i \tag{9}$$

Market clearing for labor. Each firm hires to meet demand based on the price it posted. A firm with posted price $P^{i,j}_{NT}$ must hire

$$N^{i,j} = Y^{i,j}_{NT} = \left(\frac{P^{i,j}_{NT}}{P^i_{NT}} \right)^{-\epsilon} C^i_{NT} \tag{10}$$

where C^i_{NT} is country i's aggregate nontradable demand. Labor market clearing requires that

$$N^i = \int_j N^{i,j} dj = \Delta^i_{NT} C^i_{NT} \tag{11}$$

were $\Delta_{NT} \equiv \int_j \left(\frac{P^{i,j}_{NT}}{P^i_{NT}}\right)^{-\epsilon} dj \geq 1$ is a measure of price dispersion.

Indirect utility function. Conditional on the state and realized actions earlier in the period, end-of-period equilibrium is a nominal wage W^i, a tradables price P^i_T, household quantities $\left\{C^i_T, C^{i,j}_{NT}, N^i\right\}$, firm quantities $\left\{Y^{i,j}_{NT}, \psi^{i,j}, N^{i,j}\right\}$ and a domestic government transfer t^i, such that household optimization conditions (4)-(6) are satisfied, household budgets are balanced (3), the law of one price (9) holds, firms produce to meet customer demand according to (10) and generate profits (7), the government balances its budget (8), and the labor market clears (11).

Lemma 3. *Given $\{\varphi^i\}, s, \{T^1, T^2\}, \mathcal{E}^i$, end of period equilibrium is unique. The nontradable price index and price dispersion are given by*

$$P^i_{NT} = \left(\int p^{1-\epsilon} d\varphi^i(p)\right)^{\frac{1}{1-\epsilon}}; \quad \Delta_{NT} = \int \left(\frac{p}{P^i_{NT}}\right)^{-\epsilon} d\varphi^i(p)$$

Country i consumes tradables

$$C^i_T = E^i_T(s) + T^{-i} - T^i$$

while on the nontradable side its production, consumption, and labor are given by

$$Y^i_{NT} = C^i_{NT} = \alpha \frac{\mathcal{E}^i}{P^i_{NT}} C^i_T; \quad N^i_T = \alpha \frac{\mathcal{E}^i}{P^i_{NT}} \Delta^i_{NT} C^i_T$$

The country attains indirect utility

$$v^i(\{\varphi^i\}, s, \{T^1, T^2\}, \mathcal{E}^i) = \log\left(C^i_T\right) + \alpha \left(\log\left(\alpha \frac{\mathcal{E}^i}{P^i_{NT}} C^i_T\right) - \frac{\left(\alpha \frac{\mathcal{E}^i}{P^i_{NT}} \Delta^i_{NT} C^i_T\right)^{1+\phi}}{1+\phi}\right) \tag{12}$$

If we define $V^i(h^{t-1})$ to be the expected utility of a country starting at history h^{t-1}, the following recursion holds, leaving dependence of equilibrium objects on history implicit for simplicity of notation:

$$V^i(h^{t-1}) = \sum_{s_t} \pi(s_t|s_{t-1}) \left(v^i(\{\varphi^i_t\}, s_t, \{T^1_t, T^2_t\}, \mathcal{E}^i_t) + \beta V^i(h^t)\right) \tag{13}$$

2.2.2 Central bank

Monetary authority's objective. In each period the central bank acts after observing the price distribution for nontradables $\{\varphi^i\}$, the state s, and the government transfers of $\{T^1, T^2\}$, by setting the nominal exchange rate. We consider two monetary regimes. Under independent monetary policy, country i's central bank sets the exchange rate $\mathcal{E}^i$ to maximize agent welfare in the end-of-period equilibrium:

$$\mathcal{E}^i = \arg\max_{\hat{\mathcal{E}}^i} v^i(\{\varphi^i\}, s, \{T^i\}, \hat{\mathcal{E}}^i) \tag{14}$$

Under monetary union, we assume that a unified central bank sets the common exchange rate $\mathcal{E} \equiv \mathcal{E}^1 = \mathcal{E}^2$ to maximize an equally weighted average of country welfare:

$$\mathcal{E} = \arg\max_{\hat{\mathcal{E}}} \frac{1}{2} v^1(\{\varphi^1\}, s, \{T^1, T^2\}, \hat{\mathcal{E}}) + \frac{1}{2} v^2(\{\varphi^2\}, s, \{T^1, T^2\}, \hat{\mathcal{E}}) \tag{15}$$

Beyond the natural choice of a weighted average of country welfare as an objective for the central bank in the union, the objective objective function (15) embodies two assumptions. The first one is an assumption of equal weights: this is natural given that countries have identical preferences and hence an equally-sized efficient nontradable sector (Lemma 1). The second is the assumption of a static objective. We make this assumption to prevent the central bank from becoming involved as a intertemporal player in the repeated game. It is equivalent to restricting the set of subgame perfect equilibria to exclude fiscal strategies that depend on past monetary policy. Among other things, this eliminates equilibria where the central bank uses monetary policy to punish current deviators from the fiscal union, and is itself incentivized to enforce punishments because future fiscal cooperation depends on its actions.

Ruling out equilibria where the central bank can act as a strategic enforcer of fiscal union allows us to focus on the more direct channels through which monetary and fiscal union are related. Since a primary message of this paper is that monetary union encourages fiscal risk-sharing, we view this as a conservative choice: these more elaborate equilibria only strengthen the monetary authority's role in fiscal union. Later, in Section 4, we will explore an alternative timing that allows the central bank to behave more strategically.

Stabilization. Let $\tau^i(s)$ denote the labor wedge in country i in end of period equilibrium, defined such that $1 - \tau^i(s)$ is the ratio of the marginal rate of substitution to the marginal rate of transformation between labor and aggregate nontradables:

$$1 - \tau^i(s) \equiv C^i_{NT}(s)\, \Delta^i_{NT} (N^i(s))^{\phi} \tag{16}$$

Lemma 4. *An independent central bank in country i, maximizing (14), sets*

$$\tau^i(s) = 0 \tag{17}$$

and therefore achieves the first-best in equilibrium ($C^{i,j}_{NT} = N^{i,j} = 1, \forall i, j$). The central bank in a monetary union, maximizing (15), sets

$$\frac{1}{2}\tau^1(s) + \frac{1}{2}\tau^2(s) = 0 \tag{18}$$

An independent central bank simply sets the labor wedge in its own country to 0. Since we will show in Section 2.2.4 that nontradable pricesetting results in no price dispersion ($\Delta^i_{NT} = 1$), a labor wedge of 0 is equivalent to the efficient level of nontradable consumption and production given in Lemma 1. Monetary policy then replicates the outcome that would prevail under flexible prices. The central bank in a monetary union sets an average of labor wedges to 0.

In this light, we can view objectives (14) and (15) as rules for stabilizing aggregate demand, a traditional role of monetary policy. These optimality conditions are featured by the literature on optimal monetary policy in currency unions (Benigno (2004), Galí and Monacelli (2008), Farhi and Werning (2013)).

2.2.3 Transfer policy

The government of each country i has a pure transfer strategy $T^i(h^{t-1}, (\varphi^1_t, \varphi^2_t), s_t)$, which specifies a transfer in period t conditional on the full history h^{t-1} from earlier periods, as well as the nontradable price distributions $(\varphi^1_t, \varphi^2_t)$ and exogenous state s_t realized thus far in period t. We restrict these transfers to lie in the interval $\left[0, E^i_T(s_t) - \epsilon\right]$ for some $\epsilon > 0$. This ensures compactness of the strategy set and thus that all values are finite.

In subgame perfect equilibrium, the transfer T^i in period t is set so that

$$T^i = \arg\max_{\hat{T}^i} v^i(\{\varphi^i_t\}, s_t, \{\hat{T}^i, T^{-i}\}, \mathcal{E}^i_t) + \beta V^i(h^t) \tag{19}$$

where $\{\varphi^i_t\}$ and s_t are already known, T^{-i} is given by the equilibrium strategy of the other country, $\mathcal{E}^i_t$ is given by the central bank's optimal response characterized in Lemma 4, and $V^i(h^t)$ is defined in (13). $V^i(h^t)$ implicitly incorporates the reaction of future transfers to the current decision.

Equation (19) shows that when choosing its transfer policy, the government internalizes the effect this transfer has on current indirect utility, taking into account the direct effect of the transfer on tradables consumption, as well as the indirect effect of the transfer on the nontradable side of the economy and the reaction of the central bank to the transfer. But the main tradeoff embedded in (19) is between present and future: by choosing a lower transfer $\hat{T}^i$, country i can usually improve its current utility v^i, but this may be at the expense of future utility V^i. Positive transfers on the equilibrium path are sustained by strategies that, off the equilibrium path, punish deviating countries with lower future transfers.

2.2.4 Nontradable pricesetting

Nontradable pricesetters in country i maximize expected profits (7) in end-of-period equilibrium, weighted by the stochastic discount factor of the country i household.

Lemma 5. *In each country i, in equilibrium all nontradable pricesetters j set the same price $P^{i,j}_{NT} = P^i_{NT}$, so there is no price dispersion ($\triangle^i_{NT} = 1$) and the price distribution φ^i is degenerate. The labor wedge is* *then*

$$\tau^i(s) = 1 - C^i_{NT}(s)^{1+\phi}$$

and P^i_{NT} is such that the expected labor wedge (16) in country i across all states is 0:

$$\sum_s \pi(s|s_{-1})\tau^i(s) = 0 \tag{20}$$

Note that (20), which sets the expected labor wedge for a country equal to 0, is consistent with the characterization of monetary policy in both (17)—which sets the ex-post labor wedge in the country to 0—and (18)—which sets the ex-post average of labor wedges across both countries to 0. This is necessary for equilibrium to exist, and it is due to the labor subsidy $\tau^i_L = -\frac{1}{\epsilon}$, which ensures the constrained efficiency of the monopolist's pricesetting problem. As explained in more detail in the proof of Lemma 5 (appendix A), without this subsidy, pricesetters and the central bank target inconsistent conditions, as the central bank tries to inflate away the effects of the monopolistic markup; anticipating this, pricesetters set even higher prices. Here, contrary to other models of the inflationary bias (Barro and Gordon (1983), Clarida, Galí and Gertler (1999)), there is no cost on either side from setting higher prices and this process has no fixed point unless $\tau^i_L = -\frac{1}{\epsilon}$.

2.3 Characterizing outcomes on the equilibrium path

Since the full set of subgame perfect equilibria is difficult to characterize, we first examine behavior on the equilibrium path. In Section 2.3.1, we show that given the on-path net transfers, it is possible to derive all other on-path quantities and relative prices. We follow up in Section 2.3.2 by demonstrating what we call the *risk-sharing miracle*: any transfer rule that achieves perfect risk sharing in tradable goods also achieves the first best on the nontradable side. Finally, in Section 2.3.3, we show that any on-path transfer rule attainable in subgame perfect equilibrium can be attained in an SPE of a much more specific form. This will vastly simplify the study of attainable on-path outcomes in the rest of the paper.

2.3.1 The sufficiency of net transfers

Consider any subgame perfect equilibrium. Following any exogenous history s^t, on the equilibrium path there are transfers $T^1(s^t)$ and $T^2(s^t)$. Let $T(s^t) \equiv T^1(s^t) - T^2(s^t)$ be the *net transfer* from country 1 to country 2.

Lemma 6. *Given $T(s^t)$, all quantities and relative prices on the equilibrium path are uniquely determined.*

Proof. We know from Lemma 5 that there is no price dispersion: $\Delta^i_{NT} = 1$. The characterization of end-of-period equilibrium in Lemma 3 then shows that $C^i_T(s^t)$, $C^i_{NT}(s^t)$, and $N^i(s^t)$ are uniquely determined by $T(s^t)$ and the relative prices $\mathcal{E}^i(s^t)/P^i_{NT}(s^{t-1})$, as given by the following equations:

$$C^i_T = E^i_T(s) + (-1)^i T \qquad C^i_{NT} = N^i_T = \alpha \frac{\mathcal{E}^i}{P^i_{NT}} C^i_T$$

Thus if we can show that the relative prices $\mathcal{E}^i(s^t)/P^i_{NT}(s^{t-1})$ are uniquely determined by $T(s^t)$, our result will follow.

In equilibrium, the labor wedge τ^i (16) is given as a function of $\mathcal{E}^i/P^i_{NT}$ and C^i_T by

$$\tau^i = 1 - \left(\alpha \frac{\mathcal{E}^i}{P^i_{NT}} C^i_T\right)^{1+\phi} \tag{21}$$

In the case of independent monetary policy, equation (17) shows that each country's central bank always sets the labor wedge equal to zero in every state. Given this, price-setters' optimality conditions (20) are automatically satisfied. We can then invert (21) to obtain all relative prices

$$\frac{\mathcal{E}^i(s^t)}{P^i_{NT}(s^{t-1})} = \frac{1}{\alpha\left(E^i_T\left(s^t\right) + (-1)^i T\left(s^t\right)\right)}$$

In the case of a monetary union, perfect stabilization may no longer be possible. Conditional on s^{t-1}, (18) and (20) give a set of $S+2$ equations for the labor wedges $\tau^i\left(s^{t-1}, s_t\right)$, one of which is redundant. There are $S+1$ unknown relative prices $\frac{\mathcal{E}^i\left(s^{t-1}, s_t\right)}{P^1_{NT}(s^{t-1})}$ and $\frac{P^2_{NT}\left(s^{t-1}\right)}{P^1_{NT}(s^{t-1})}$, matching the number of nonredundant equations. The proof in appendix A shows there always exists a unique solution for these relative prices.

Note that always is some nominal indeterminacy. In the independent monetary policy case, each country can have its own price level in every period; in the monetary union the overall price level is undetermined in every period. Such indeterminacy is the result of our assumption that prices are reset in every period, and has no allocative consequences.

□

2.3.2 Risk-sharing miracle

Even though perfect stabilization is generally not feasible in monetary union, there is an important special case in which it is.

Theorem 1 (Risk-sharing miracle)**.** *If in period t, the net transfers $T\left(s^t\right)$ achieve first-best risk sharing across all states s_t, the first best is also achieved for the nontradable side, even in monetary union.*

Proof. Under independent monetary policy, the first best is always achieved (Lemma 4). Under monetary union, the first best in country i requires that

$$\tau^i\left(s^t\right) = 1 - \frac{\mathcal{E}\left(s^t\right)}{P^i_{NT}\left(s^{t-1}\right)} C^i_T\left(s^t\right) = 0 \tag{22}$$

First-best tradable risk sharing achieves $\frac{C^2_T\left(s^t\right)}{C^1_T\left(s^t\right)} = \lambda$ for some constant λ. Relative prices

$$\frac{P^2_{NT}\left(s^{t-1}\right)}{P^1_{NT}\left(s^{t-1}\right)} = \lambda \quad \text{and} \quad \frac{\mathcal{E}\left(s^t\right)}{P^1_{NT}\left(s^{t-1}\right)} = \frac{1}{C^1_T\left(s^t\right)}$$

then imply

$$\frac{\mathcal{E}\left(s^t\right)}{P_{NT}^2\left(s^{t-1}\right)} = \frac{1}{C_T^1\left(s^t\right)}\frac{1}{\lambda} = \frac{1}{C_T^2(s^t)}$$

At those prices, (22) is satisfied simultaneously in both countries. With the labor wedge equal to zero in both countries and in all states, the equilibrium conditions for monetary policy (18) and price-setting (20) are then trivially satisfied. □

The intuition behind the risk-sharing miracle is that when countries share risks appropriately through fiscal policy, they make the appropriate stance of monetary policy identical across countries. The central bank, by setting its policy instrument as an average of the desirable level for each country, is therefore able to stabilize them both simultaneously. In this way, the risk-sharing miracle is a sharp illustration of the longstanding view that closer fiscal union reduces the difficulties created by a common currency.

2.3.3 Implementation using grim-trigger equilibria

Since we are interested in attainable on-path outcomes for quantities and relative prices, our analysis will be facilitated by the result in this Section, which shows that such outcomes can be implemented by a grim-trigger strategy.

Definition 1. A grim-trigger strategy that sustains a given net transfer rule $T\left(s^t\right)$ specifies that if net transfers $T\left(s^\tau\right)$ have been observed for all $s^\tau \prec s^t$, countries make transfers

$$T^1\left(h^t\right) = \max\left\{T\left(s^t\right), 0\right\} \quad T^2\left(h^t\right) = \max\left\{-T\left(s^t\right), 0\right\}$$

and otherwise, they each make transfer $T^i\left(h^t\right) = 0$.

For the off-path permanent choice of $T^i\left(h^t\right) = 0$ to be part of a subgame perfect equilibrium, the choice of $T^i = 0$ must constitute a Nash equilibrium of the stage game. Although this will generally be the case, in our environment it is in principle possible for countries to be in such extreme booms that they find unilateral transfers preferable to autarky, because these transfers lead to decreased demand for nontradable goods and relieve the pressure on the domestic economy. We rule this possibility out with the following assumption:

Assumption 3 (No voluntary unilateral transfer in autarky)**.** *Parameters are such that, for countries living in autarky within a monetary union, it is never desirable to make unilateral transfers.*

Appendix A provides the assumption on primitives to which assumption 3 is equivalent. It also provides a simpler and stronger sufficient condition: when the countries are relatively open, in the sense that

$$\alpha < \frac{8}{1 + \max_s\left\{\frac{E_T^1(s)}{E_T^2(s)}; \frac{E_T^2(s)}{E_T^1(s)}\right\}} \tag{23}$$

neither country ever wants to make unilateral transfers in autarky, irrespective of the way nontradables prices were set.

Assumption 4 (Ex-ante option to withdraw)**.** *At the beginning of each period, countries can commit not to make any outgoing transfer and to refuse any incoming transfer.*

Although countries cannot commit to making any particular level of transfer, assumption 4 imparts them with a small level of commitment, intended to rule out the possibility of an expectations trap where self-sustaining transfers arise only because price-setters expect them to happen, delivering lower utility to countries than what they could get if they lived in autarky forever. The following result follows from assumptions 3 and 4:

Lemma 7. *In monetary union, the autarky allocation is subgame perfect and provides the lowest utility level to both agents of any subgame-perfect equilibrium.*

Proof. Since transfers are restricted to lie in $\left[0, E^i_T(s) - \epsilon\right]$, the set of values achievable in SPE is bounded, so it has minimum M. Consider any subgame perfect equilibrium that attains the ex-ante value V. By assumption 4, a permanent deviation that attains the flow value of autarky in each period is always available, delivering V^{aut}, so $V \geq V^{aut}$. As a consequence of assumption 3, autarky is a static Nash equilibrium, so its infinite repetition is subgame perfect, showing $V^{aut} \geq M$. Hence $V \geq V^{aut} \geq M$ for any value V attained by a subgame-perfect equilibrium, and in particular for the minimal value M, showing that $V^{aut} = M$. □

We conclude with this Section's main theorem.

Theorem 2. *Any net transfer rule $T\left(s^t\right)$ attainable in subgame perfect equilibrium is also attainable in an SPE where countries follow grim-trigger strategies.*

Proof. Consider a subgame perfect equilibrium. By definition of subgame perfection, the value $V^i\left(h^t\right)$ attained on path for country i after history h^t is higher than the value of any possible deviation: $V^i\left(h^t\right) \geq V^{dev,i}\left(h^t\right)$. Since any deviation is itself subgame perfect, from Lemma 7, in turn we have $V^{dev,i}\left(h^t\right) \geq V^{aut,i}\left(s^t\right)$ after every history. As in Section 2.3.1, denote by $T\left(s^t\right)$ the net transfer from country 1 to country 2 that arises on the equilibrium path. Consider then replacing the subgame perfect equilibrium with a grim trigger strategy sustaining the transfer rule $T\left(s^t\right)$. By Lemma 6, this strategy delivers the same on-path equilibrium outcomes, and so delivers the same value $V^i\left(s^t\right)$ on the equilibrium path. By the above argument, we therefore have

$$V^i\left(s^t\right) \geq V^{aut,i}\left(s^t\right) \quad \forall i,\ \forall s^t \tag{24}$$

Since the considered grim-trigger strategy delivers $V^{aut,i}\left(s^t\right)$ after any deviation, (24) shows that this strategy constitutes a subgame-perfect equilibrium that delivers the same net transfer rule $T\left(s^t\right)$ as the initial SPE. □

Theorem 2 follows the traditional approach to repeated games, where sustainable outcomes are characterized using the worst off-path punishment (Abreu (1988)). It shows that in our complicated game, the worst punishment is still autarky, just as in the traditional limited commitment literature (see for example Kocherlakota (1996)).

3 Risk-sharing benefits of monetary union

In this section, we develop our main results using a symmetric structure for endowments and transfer strategies.

Assumption 5. *Countries' endowment processes are ex-ante symmetric, as follows. There exists a finite-state Markov process z_t with elements $z_t \in \mathbf{Z}$ and transition matrix Π^z, and a pair of endowment levels $E^H(z) \geq E^L(z)$ for each $z \in \mathbf{Z}$. Countries' endowment processes E^i are such that*

$$\Pr\left(E^i = E^H(z_t)\,|z^t\right) = \Pr\left(E^i = E^L(z_t)\,|z^t\right) = \frac{1}{2} \quad i \in \{1,2\}$$

This process allows an arbitrary degree of persistence in both the level and the volatility of union-wide tradable output, but imposes that countries' relative fortunes have an equal chance of being reversed in every period.

Definition 2. A Markov transfer rule is a function $T(z|z_{-1})$ that specifies the transfer from the country with endowment $E^H(z)$ to the country with endowment $E^L(z)$ at any $z \in \mathbf{Z}$ following $z_{-1} \in \mathbf{Z}$.

When restricting endowments to be ex-ante symmetric and countries' fiscal arrangements to Markov transfer rules, the analysis of ex-ante price setting is simplified dramatically, as the following Lemma illustrates.

Lemma 8. *When countries have endowment processes governed by assumption 5 and when they follow Markov transfer rules, their relative nontradable prices are always equal under monetary union:*

$$\frac{P^1_{NT}\left(s^{t-1}\right)}{P^2_{NT}\left(s^{t-1}\right)} = 1 \quad \forall s^{t-1}$$

In particular, we can normalize both these prices to 1. This consequence of symmetry allows us to cut through the complexity imposed by the relative price-setting decisions of producers in each country. It guarantees that the real exchange rate, $\frac{P_T\left(s^t\right)}{P_{NT}\left(s^{t-1}\right)} = \frac{\mathcal{E}\left(s^t\right)}{P_{NT}\left(s^{t-1}\right)}$ is the *same* in both countries of the monetary union at any point in time. This allows us to provide sharp comparisons of the feasible degree of risk-sharing under independent monetary policy and monetary union, respectively.

3.1 Improved risk-sharing under monetary union

Definition 3. A Markov transfer rule with some risk sharing is a Markov transfer rule $T(z|z_{-1})$ such that $T(z|z_{-1}) \in \left[0, \frac{E^H(z)+E^L(z)}{2}\right]$ for all $z, z_{-1} \in \mathbf{Z}$.

Under Markov transfer rules with some risk sharing, endowments and tradable consumption levels are ordered as

$$E^L(z) \leq C^L_T(z) \leq C^H_T(z) \leq E^H(z) \tag{25}$$

Theorem 3. *Any Markov transfer rule with some risk sharing that is achievable in SPE under independent monetary policy is also achievable in SPE in a monetary union.*

Proof. Under independent monetary policy, countries' nontradable sides are always at their efficient level in every period (Lemma 4). Consider a Markov transfer rule with some risk sharing achievable under this monetary regime. By Theorem 2, the same transfer rule is achievable under an SPE that reverts to autarky following any deviation. By definition of subgame perfection, the H country does not want to refrain from the transfer at any node. Using the Markov structure, there is such a participation constraint for every $z \in \mathbf{Z}$, expressed as

$$\underbrace{\log\left(E_T^H(z)\right) - \log\left(C_T^H(z)\right)}_{\text{One-shot gain from defaulting}}$$
$$\leq \underbrace{\beta \sum_{z} \frac{\tilde{\pi}(z'|z)}{2}\left[\left(\log\left(C_T^L(z')\right) - \log\left(E^L(z')\right)\right) - \left(\log\left(E^H(z')\right) - \log\left(C_T^H(z')\right)\right)\right]}_{\text{Expected loss from lack of future risk-sharing}} \tag{26}$$

where the probabilities $\tilde{\pi}(z'|z)$, given in the appendix, take into account the relevant mix of future probabilities and discounting. Given concavity of the log function and (25), the loss of future risk sharing that comes from reversion to autarky entails costs, and these costs overwhelm the one-off gains from leaving the union when (26) is satisfied.

Consider sustaining the same SPE under monetary union using the same on-path and off-path actions. Lemma 8 implies that both countries have the same real exchange rate at every node, so they evaluate tradable consumption levels using the same indirect utility function

$$\tilde{v}_z(C_T) = \log(C_T) + \alpha\left(\log(\alpha\epsilon_z(C_T)C_T) - \frac{1}{1+\phi}(\alpha\epsilon_z(C_T)C_T)^{1+\phi}\right)$$

where $\epsilon_z(C_T)$ is the equilibrium reaction of the central bank to the level of tradable consumption C_T when the state is z, given in appendix A. If we can check that a version of (26) holds with log replaced by $\tilde{v}_z$, this guarantees that the participation constraint for the H country is met in every state z under monetary union. Since the L participation constraint is trivially satisfied at every node given that $E^L(z) \leq C_T^L(z)$, $T(z|z_{-1})$ is indeed sustainable in a monetary union SPE and the theorem follows.

Appendix A gives a formal argument, but here we illustrate why (26) does hold with the indirect utility function $\tilde{v}_z$. Figure 3 illustrates that the costs from less future risk-sharing are greater: by the risk-sharing miracle, a rule that delivers perfect risk-sharing $T(z') = \frac{E^H(z')+E^L(z')}{2}$ at every future node attains the same utility for the country as it does under independent monetary policy, but deviations are now more costly because of the macroeconomic externalities associated with the central bank's inability to perfectly stabilize. Figure 3 also illustrates that the benefits of leaving are smaller: a country tempted to leave the fiscal union is already overheated, and leaving the union exacerbates this boom.

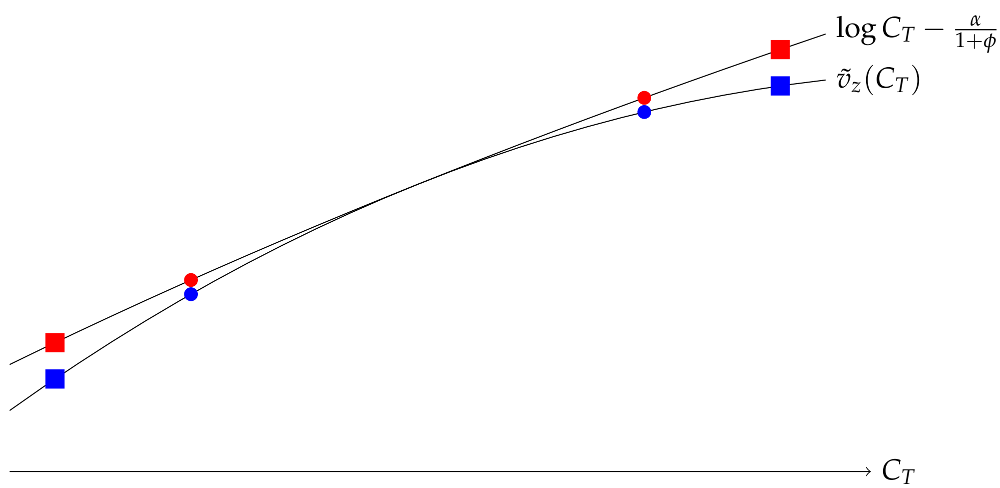

Figure 3: Costs and benefits of participating in fiscal union under alternative monetary regimes

□

3.2 An example of powerful improvement

We now show that the risk-sharing incentives can improved so much under monetary union as to transport countries from autarky to first-best.

Theorem 4. *There exists a combination of parameters and endowment processes such that autarky is the only feasible risk-sharing outcome when countries run an independent monetary policy, but a SPE with full risk-sharing is possible under monetary union.*

Proof. We consider the simplest possible example of our framework: the symmetric iid case with union-wide tradable output equal to 1. In the notation above, the Markov chain is reduced to a point $z = 1$ in every period, $E^H = e$ and $E^L = 1 - e$, with $e > \frac{1}{2}$. In this context, a Markov transfer is a value T, such that $\left(C_T^L, C_T^H\right) = (1 - e + T, e - T)$. We consider transfers that improve risk-sharing, in other words $T \in \left[0, \frac{1}{2} - e\right]$.

Appendix A demonstrates formally the following propositions. Under independent monetary policy, our setup collapses to a simple limited commitment model. Some risk sharing ($T > 0$) is feasible if the country currently in the high state is patient enough to value the benefits from future reciprocity: its discount factor must be above a lower bound, $\underline{\beta}^{indep} = 2\,(1 - e)$. Conversely, a deviation from first-best risk sharing, where $T = \frac{1}{2} - e$, is valuable if its discount factor is below an upper bound $\overline{\beta}^{indep}$, derived from the participation constraint of the country with endowment e.

Consider now the case of a monetary union, and assume countries are achieving perfect risk-sharing $T = \frac{1}{2} - e$. Due to the risk-sharing miracle (theorem 1), their nontradables side is perfectly stabilized, so their values from the transfer arrangement are identical to those under independent monetary policy. However, a deviation now entails an additional cost $c\,(\alpha, \phi, e)$ coming from the fact that the defaulting country will experience a macroeconomic boom. Going forward, the central bank loses its ability to stabilize both countries simultaneously, which creates additional utility costs $c(\alpha, \phi, e)$ and $c(\alpha, \phi, 1-e)$ in each state. This implies that the discount factor threshold above which full risk-sharing is feasible is now $\overline{\beta}^{union}(\alpha, \phi, e)$, a function of parameters governing the nontradable side.

Figure 4 illustrates these thresholds, for an illustrative calibration with $e = 0.7$ and $\phi = 1$. When $\alpha = 0$, the nontradable side is inexistent and the discount factor threshold for first-best under monetary union and independent monetary policy coincide: $\overline{\beta}^{indep} = \overline{\beta}^{union}(0, \phi, e)$. Countries with discount factors below $\underline{\beta}^{indep} = 0.6$ cannot sustain any risk-sharing under independent monetary policy, countries with discount factors above $\underline{\beta}^{indep}$ can sustain first-best, and countries with intermediate discount factors can sustain some, but not full risk-sharing. As α grows and countries nontradables side becomes more important, it becomes easier to sustain risk-sharing. For α around 2.5, countries with discount factors around 0.5 cannot not sustain *any* risk-sharing under independent monetary policy but can sustain full risk-sharing under monetary union. If α becomes too large ($\alpha \geq \overline{\alpha}(e, \phi)$), the autarky punishment is no longer subgame perfect: the country high state is in such a boom under autarky that it prefers to make a unilateral transfer to cool its tradable side. □

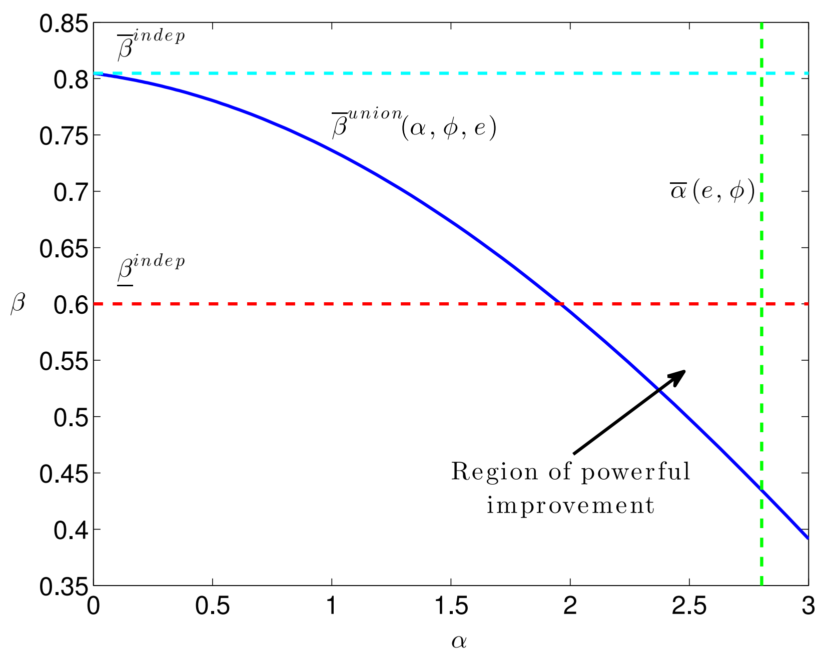

Figure 4: Illustration of thresholds for theorem 4

3.3 Discussion

The theorems in this section are two facets of our claim that monetary union begets fiscal union. By specializing the framework of Section 2 to a case where endowments and transfer rules have limited history dependence, we are able to prove in theorem 3 that fiscal union can improve risk sharing in a particularly clear sense: any transfer rule that was feasible under independent monetary policy is still feasible under monetary union. And theorem 4 shows that it is possible to find powerful improvements in this class of equilibria. We now discuss the generality of these results, by considering what would happen if we relaxed some of the assumptions imposed.

Consider relaxing the assumptions on symmetry of endowments and transfer rules. The modern literature on limited commitment (Kocherlakota (1996), Alvarez and Jermann (2000), Ligon et al. (2002)) emphasizes that it is in general possible to sustain subgame-perfect outcomes that improve upon Markov transfer rules. In the equilibria characterized by this literature, the amount a country owes depends not only on its current and previous state, but on the full history of past shocks, which an endogenous state variable (promised utility) keeps track of. For this more general class of equilibria, theorem 3 is in general no longer true. In particular, the country hitting its participation constraint is no longer unambiguously the country which would experience a boom if if left the union: a country with a history of very bad shocks may be held at its participation constraint as it is called upon to pay back in a mild state, even if its endowment is still relatively low. However, even under this class of equilibria, there is still a sense in which risk-sharing is ameliorated under monetary union: the discount factor thresholds to attain first-best are ordered $\overline{\beta}^{union} \leq \overline{\beta}^{indep}$. In fact, it is generally possible to find powerful improvements as in theorem 4 for these more general endowment structures. Because of the risk-sharing miracle, the optimal policies in the fiscal union involve first-best risk sharing, which is simple to characterize.

Another way to relax assumptions is to add shocks to the nontradable side of the economy. Such shocks can be modelled in our framework by assuming that preferences for nontradables are dependent on the exogenous state: $\alpha^i(s)$. In this case, the risk-sharing miracle is in general no longer true, as can be seen by the following argument. Assume that countries share risks to tradables perfectly, so that their relative tradables consumption is constant across all states. Since under monetary union they share the same nominal exchange rate, their relative nontradable consumption in a state s is then, from households' first-order condition,

$$\frac{C^1_{NT}(s)}{C^2_{NT}(s)} = \lambda \frac{\alpha^1(s)}{\alpha^2(s)} \tag{27}$$

where λ is a constant reflecting the risk-sharing rule and nontradable prices that are constant across all states. Unless α^1 and α^2 vary proportionally across states, (27) is incompatible with efficient consumption of nontradables, which still requires that $C^1_{NT}(s) = C^2_{NT}(s) = 1$ (Lemma 8). The constrained-efficient outcome that takes into account nominal rigidities, which fiscal union would reach absent the limited commitment constraint, does not feature perfect stabilization in each country (Farhi and Werning (2013)). This means that joining a monetary union necessarily entails some welfare losses from imperfect stabilization, but even in this case, there is still a force pushing for welfare gains from improved incentives to share risks, so the overall welfare benefit from transiting into monetary union might be positive. In this sense the overall message of the model — the risk-sharing benefits of monetary union have to be balanced against the stabilization costs — is unchanged by the presence of shocks to nontradables.

4 Optimal joint monetary and fiscal policy in the union

4.1 Alternative timing and the role of monetary policy

In Sections 2 and 3, we assumed that the central bank sets the exchange rate after countries announced their transfers. With static welfare maximization as its objective, the central bank was limited to stabilizing the aggregate economy ex post, without any commitment power or ability to internalize the sustainability of fiscal union. This assumption allowed us to evaluate the direct effects of monetary union, without considering the central bank as a strategic actor in its own right—which is inevitably a more speculative exercise.

In this section, we broaden the role of monetary policy, allowing the central bank in a monetary union to commit to an exchange rate policy at the beginning of each period, while retaining its objective of within-period welfare maximization. We now replace the timing from Figure 2 with that depicted in Figure 5:

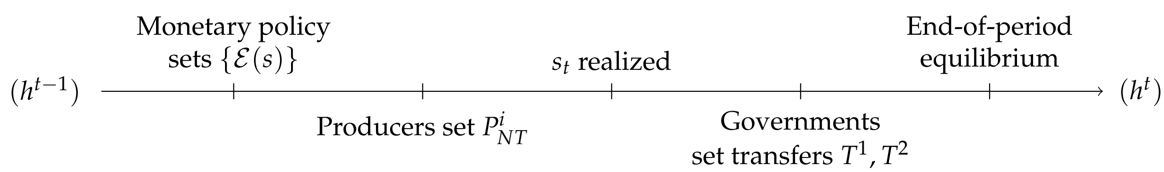

Figure 5: Alternative Timing

The choice of a state-contingent $\{\mathcal{E}(s)\}$ at the beginning of the period is driven by expected welfare maximization

$$\mathcal{E}(s) = \arg\max_{\{\hat{\mathcal{E}}(s)\}} \sum_s \pi(s) \left(\frac{1}{2} v^1(\{\varphi^1\}, s, \{T^1(s), T^2(s)\}, \hat{\mathcal{E}}(s)) + \frac{1}{2} v^2(\{\varphi^2\}, s, \{T^1(s), T^2(s)\}, \hat{\mathcal{E}}(s)) \right) \quad (28)$$

where the dependence of price distributions $\{\varphi^i\}$ and transfers $T^i(s)$ on monetary policy (through the reaction functions of nontradable pricesetters and governments) in (28) is left implicit.

Since the central bank moves first, it can internalize the effect of its decision on governments' incentives to make transfers, and it will not necessarily find aggregate stabilization optimal—thus overturning the result from Lemma 4. It may instead devise policy that actively encourages sustained fiscal union, expanding upon the complementarity between monetary and fiscal union derived in Section 3.

4.2 Expansionary monetary policy and aggregate dispersion

To illustrate the role of monetary policy in this new environment, we specialize Assumption 5 to a simpler case where the stochastic process for endowments is iid across periods, and symmetric within each period.

Assumption 6. *There exist finitely many $z \in \mathbf{Z}$, each of which is associated with a probability $\pi(z)$ and a pair of endowment levels $E^H(z) \geq E^L(z)$. Endowments are iid across periods, and in each period are drawn such that for each z,*

$$\Pr\left(E^1 = E^H(z) \text{ and } E^2 = E^L(z)\right) = \Pr\left(E^1 = E^L(z) \text{ and } E^2 = E^H(z)\right) = \frac{1}{2}\pi(z)$$

We will characterize the optimal relationship between the stance of monetary policy and the distribution of endowments across states. As in Section 3, we consider equilibria with symmetric strategies that depend only on the current state (and whether there has yet been a deviation), rather than depending on the full history of past actions. We also repeat Assumption 3 by ruling out extreme cases where the boom in a country is so great that a unilateral transfer is worthwhile.

Assumption 7. *Consider equilibria where the government receiving endowment H makes transfer $T(z;d)$, where $z \in \mathbf{Z}$ is the aggregate state and $d \in \{0,1\}$ is an indicator specifying whether play is on or off the equilibrium path. Also restrict attention to equilibria where unilateral transfers are never worthwhile.*

Observe that the restriction on strategies in Assumption 7, along with the symmetry of the endowment process, ensures, just as in Lemma 8, that price-setters in both countries set the same nontradable price. We can again normalize this price to 1: $P^1_{NT} = P^2_{NT} \equiv 1$.

Now suppose that, out of all the equilibria consistent with Assumption 7, we aim to characterize the equilibrium with the highest expected welfare. Quantities in this equilibrium must solve the following planning problem:

$$\max \quad \sum_z \pi(z)\left(w(E^H_T(z) - T(z), p(z)) + w(E^L_T(z) + T(z), p(z))\right) \tag{29}$$

$$\text{s.t.} \quad w(E^H_T(z), p(z)) - w(E^H_T(z) - T(z), p(z)) \leq V \quad \forall z \tag{30}$$

$$\sum_z \pi(z) \cdot \frac{\tau^H(z) + \tau^L(z)}{2} = 0 \tag{31}$$

where $p(z) \equiv P_T(z)/P_{NT} = 1$ is the relative price of tradables and nontradables in state z, $w(C_T, p) \equiv \log C_T + \alpha\left(\log(\alpha p C_T) - \frac{(\alpha p C_T)^{1+\phi}}{1+\phi}\right)$ is the indirect utility function corresponding to C_T, and V is the difference between expected future welfare along the equilibrium path and expected future welfare following a deviation. (30) is simply the participation constraint, which is necessary to ensure that the government with the high endowment makes transfer $T(z)$ rather than deviating and hoarding its entire endowment; while (31) is imposed by nontradable price-setting, following (20) in Lemma 5.

In the previous environment, Lemma 4 showed that the central bank stabilizes the aggregate economy of the monetary union, setting the average labor wedge across both countries to zero. More generally, the average labor wedge summarizes the nontradable side of the union economy: a negative labor wedge corresponds to an aggregate boom, while a positive labor wedge corresponds to an aggregate bust. In an optimum equilibrium that solves the planning problem (29)-(31), the central bank no longer seeks stabilization in every state. Instead, there is a remarkably simple relationship between macroeconomic conditions and dispersion $E^H(z)/E^L(z)$ of the endowments, captured in the following theorem.

Theorem 5. *Consider the endowment process given by Assumption 6 and equilibria as described by Assumption 7. In the subgame perfect equilibrium with maximal expected welfare, $(\tau^H(z) + \tau^L(z))/2$ is weakly decreasing in the dispersion $E^H(z)/E^L(z)$ between endowments. It is strictly decreasing in $E^H(z)/E^L(z)$ for any z such that risk-sharing is neither perfect ($C_T^H(z) = C_T^L(z)$) nor absent ($C_T^H(z) = E^H(z)$).*

In other words, in the optimal equilibrium, the central bank *creates booms when dispersion is high*. The intuition for this result is simple: when endowments are more dispersed, risk sharing is more important, and the country with a high endowment will be more willing to make a transfer when it is experiencing more of a boom. The central bank is willing to accept the cost of a partly overheated union-wide economy in order to create this boom and facilitate risk sharing in the fiscal union.

More concretely, consider the participation constraint (30), with the indirect utility function w expanded:

$$(1+\alpha)\left(\log(E_T^H(z)) - \log(E_T^H(z) - T(z))\right) \\ - \frac{\alpha}{1+\phi}\alpha^{1+\phi}p(z)^{1+\phi}\left(E_T^H(z)^{1+\phi} - (E_T^H(z) - T(z))^{1+\phi}\right) \leq V \quad (32)$$

Given the stipulation in Assumption 7 that unilateral transfers are not optimal, the left side of (32) must be increasing in $T(z)$; when the transfer is larger, making a transfer is less desirable relative to autarky. Since higher $p(z)$ decreases the total value on the left and relaxes the constraint, it enables higher transfers. Hence when transfers are particularly valuable—as in cases of high dispersion—it is worthwhile to raise $p(z)$ to the point where there is an aggregate boom, trading off the (initially) second-order costs of aggregate overexpansion against the first-order benefits of better risk sharing.

This is depicted graphically in Figure 6, which plots the welfare $w(C_T, p)$ of a country with consumption C_T and relative price p for two different values of p, the price $p = p'$ that achieves aggregate stabilization and the higher price $p = p''$ that creates an aggregate boom. Squares depict endowments, while circles depict consumption after transfers. When the central bank raises the relative price from p' to p'', at high levels of consumption there is even more of a boom, attenuating the within-period welfare loss from making a transfer and leaving the high-endowment country's participation constraint easier to satisfy. The change in monetary policy causes a first-order decrease in welfare for the booming high-endowment country and a first-order increase in welfare for the depressed low-endowment country, netting out to only a second-order loss for the union as a whole when p'' is close to the stabilization level p'. At the margin, this loss is offset by the benefits from easing the participation constraint.

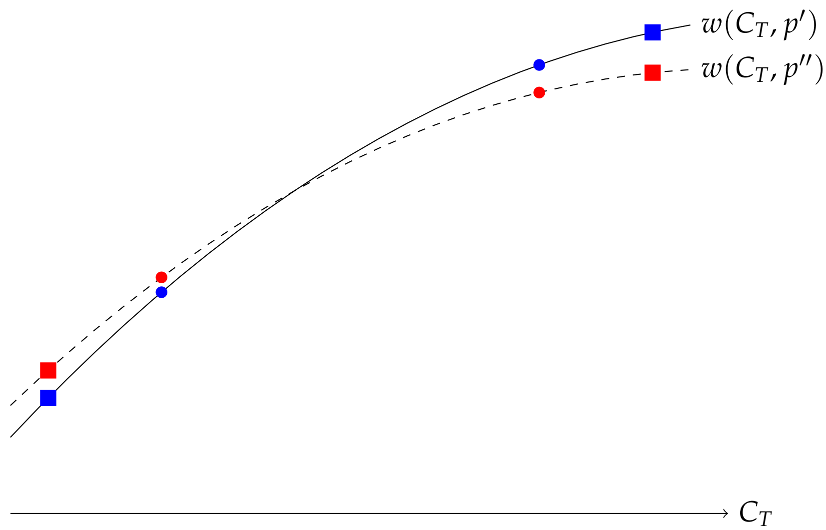

Figure 6: Welfare and transfer incentives under different monetary policies

The logic of Figure 6 suggests that the central bank should create a boom whenever the participation constraint is binding—which will generally be the case when perfect risk sharing is not achieved. Of course, it is not possible for the central bank to create a boom in every state: the nontradable sector sets prices that achieve aggregate stabilization in expectation, as indicated by (31). The central bank can only shape the *relative* pattern of boom and bust across states, not create booms across the board—and, as Theorem 5 finds, in the best equilibrium it chooses to create booms in states with more dispersed endowments, when it is particularly important to loosen participation constraints and encourage transfers.

5 Conclusion

In this paper, we have examined how monetary union facilitates the formation of a stronger fiscal union. We have seen, in an important special case, how any risk-sharing arrangement that can be sustained under independent monetary policy can also be sustained under monetary union. In fact, dramatic improvement is possible: even when no risk-sharing is possible in equilibrium with independent policy, monetary union can sometimes bring governments to a first-best outcome.

The key force is the complementarity of monetary and fiscal union: when countries more effectively share risks, their outcomes are more closely aligned, and a central bank that stabilizes the union-wide economy can come closer to stabilizing each individual economy as well. This not only makes fiscal union a desirable counterpart to monetary union—as is widely understood—but also provides a channel through which monetary integration can enable otherwise unsustainable risk-sharing. Without independent monetary policy as a fallback, governments have a greater stake in preserving joint fiscal arrangements. This bears out the progression from monetary to fiscal union envisaged in the literature on sequencing theory; and it suggests a possible upside to a common currency, when most models featuring nominal rigidities offer only drawbacks.

Further exploring the role of monetary policy in a fiscal union, we found that when a union-wide central bank behaves strategically—taking the sustainability of the fiscal union into account—the optimal rule is expansionary when dispersion within the monetary union is high. Booms make governments more willing to provide transfers, and these transfers are most important when outcomes vary greatly within the union. A singleminded emphasis on stabilization is not optimal.

To what extent are the forces in this paper visible in practice? Certainly the Euro Area today is far from perfect risk sharing—the willingness of the core to subsidize the periphery has clear limits. At the same time, the level of cross-country support under monetary union, although often frustratingly limited, has greatly exceeded what came before. There have been multiple bailouts and transfer schemes—both explicit and implicit, often taking the form of below-market lending—made with the explicit intent of preserving the common currency's viability.

Our model offers other reasons for guarded optimism. Although we show in Theorem 3 that monetary union expands the set of attainable risk sharing equilibria, we cannot be sure that governments will immediately take advantage of this feature by coordinating on the better equilibria. (After all, autarky is always an equilibrium as well.) But as participants become aware of the heightened importance of fiscal union when exchange rates are no longer free to adjust, they may learn to play the equilibrium with improved risk sharing.

Monetary policy also has an important role. In recent years, dispersion in Euro Area outcomes has coincided with the deepest aggregate slump in decades. This is exactly the opposite of the optimal arrangement in Theorem 5, which prescribes monetary accommodation that creates a boom whenever members' fates diverge and the fiscal union is under stress. To some extent, this inconsistency may be due to limitations on monetary policy that this model leaves out—in particular, the zero lower bound. But the insufficiently expansionary policy in the Euro Area is also partly by choice: the ECB raised rates in 2011, just as the fiscal prospects of peripheral countries were rapidly deteriorating, and it has since been hesitant to employ unconventional expansionary policy. Our model implies that for the full promise of fiscal union to be realized, very different choices are needed from monetary policymakers.

Monetary union begets fiscal union—but not necessarily overnight.

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A Appendix: proofs of theorems

Proof of Lemma 1. Substituting production into preferences

$$\log\left(C_T\right) + \alpha\left(\frac{\epsilon}{\epsilon-1}\log\left(\int_j \left(C_{NT}^j\right)^{\frac{\epsilon-1}{\epsilon}} dj\right) - \frac{\left(\int_j C_{NT}^j dj\right)^{1+\phi}}{1+\phi}\right)$$

and taking a first-order condition with respect to C_{NT}^k shows that

$$\frac{\left(C_{NT}^k\right)^{-\frac{1}{\epsilon}}}{\int_j \left(C_{NT}^j\right)^{\frac{\epsilon-1}{\epsilon}} dj} = \left(\int_j C_{NT}^j dj\right)^{\phi} \quad \forall k$$

This shows that the efficient allocation is constant consumption across goods, $C^k_{NT} = C^*_{NT}\ \forall k$, and further that C_{NT} satisfies

$$\frac{1}{C_{NT}} = C^{\phi}_{NT} \quad \Rightarrow \quad C^*_{NT} = 1$$

□

Proof of Lemma 4. Define the function $C^i_{NT}(e) = \alpha \frac{e}{P^i_{NT}} C^i_T$, and note that $\frac{dC^i_{NT}(e)}{de} = \alpha \frac{C^i_T}{P^i_{NT}} = \frac{C^i_{NT}(e)}{e}$. Under independent monetary policy, the central bank chooses the exchange rate e to maximize

$$v^i(e) = \log\left(C^i_T\right) + \alpha\left(\log\left(C^i_{NT}(e)\right) - \frac{\left(\Delta^i_{NT} C^i_{NT}(e)\right)^{1+\phi}}{1+\phi}\right)$$

Its first order condition is

$$\alpha \frac{C^i_{NT}(e)}{e}\left(\frac{1}{C^i_{NT}(e)} - \left(\Delta^i_{NT}\right)^{1+\phi}\left(C^i_{NT}(e)\right)^{\phi}\right) = 0$$

resulting in

$$1 - \left(\Delta^i_{NT} C^i_{NT}\right)^{1+\phi} = 0$$

combining the definition of the labor wedge in (16) with labor market clearing (11), we obtain

$$\tau^i = 1 - C^i_{NT}\Delta^i_{NT}\left(N^i\right)^{\phi} = 1 - \left(C^i_{NT}\Delta^i_{NT}\right)^{1+\phi} = 0$$

The central bank's exchange rate choice $\mathcal{E}^i \equiv e$ is given by

$$\mathcal{E}^i = \frac{1}{\alpha}\frac{P^i_{NT}}{\Delta^i_{NT} C^i_T} \tag{33}$$

Under joint monetary policy, the central bank chooses e to maximize

$$\frac{1}{2}v^1(e) + \frac{1}{2}v^2(e)$$

resulting in the first-order condition

$$\begin{aligned}
\frac{1}{2}\left(\frac{C^1_{NT}}{e}\right)\left(\frac{1}{C^1_{NT}} - \left(\Delta^1_{NT}\right)^{1+\phi}\left(C^1_{NT}\right)^{\phi}\right) + \frac{1}{2}\left(\frac{C^1_{NT}}{e}\right)\left(\frac{1}{C^2_{NT}} - \left(\Delta^2_{NT}\right)^{1+\phi}\left(C^2_{NT}\right)^{\phi}\right) &= 0 \\
\frac{1}{2}\left(1 - \left(\Delta^1_{NT}C^1_{NT}\right)^{1+\phi}\right) + \frac{1}{2}\left(1 - \left(\Delta^2_{NT}C^2_{NT}\right)^{1+\phi}\right) &= 0 \\
\frac{1}{2}\tau^1 + \frac{1}{2}\tau^2 &= 0
\end{aligned}$$

which is equation (18). That is, the central bank's exchange rate choice $\mathcal{E}$ solves

$$\frac{1}{2}\left(\Delta_{NT}^{1}\alpha\frac{\mathcal{E}}{P_{NT}^{1}}C_{T}^{1}\right)^{1+\phi}+\frac{1}{2}\left(\Delta_{NT}^{2}\alpha\frac{\mathcal{E}}{P_{NT}^{2}}C_{T}^{2}\right)^{1+\phi}=1 \tag{34}$$

resulting in

$$\mathcal{E}=\frac{1}{\alpha}\left(\frac{1}{2}\left(\frac{P_{NT}^{1}}{\Delta_{NT}^{1}C_{T}^{1}}\right)^{-(1+\phi)}+\frac{1}{2}\left(\frac{P_{NT}^{2}}{\Delta_{NT}^{2}C_{T}^{2}}\right)^{-(1+\phi)}\right)^{-\frac{1}{1+\phi}} \tag{35}$$

□

Proof of Lemma 5. Using the relationship between consumer nontradable and tradable demand (4), we can rewrite firm profits (7) given price p, when the nominal wage is $W^{i}(s)$, the nominal exchange rate is $\mathcal{E}^{i}(s)$, and the price index for nontradables is $P_{NT}^{i}(s)$, as

$$\psi^{i,j}(p)=\left(p-\left(1+\tau_{L}^{i}\right)W^{i}(s)\right)\left(\frac{p}{P_{NT}^{i}(s)}\right)^{-\epsilon}\alpha\frac{\mathcal{E}^{i}(s)}{P_{NT}^{i}(s)}C_{T}^{i}(s) \tag{36}$$

Firms maximize the expected level of (36), valuing nominal profits in each state using the nominal stochastic discount factor $\frac{1}{\mathcal{E}^{i}(s)C_{T}^{i}(s)}$:

$$\begin{aligned}\Pi(p) &= \sum_{s}\pi(s|s_{-1})\frac{1}{\mathcal{E}^{i}(s)C_{T}^{i}(s)}\left(p-\left(1+\tau_{L}^{i}\right)W^{i}(s)\right)\left(\frac{p}{P_{NT}^{i}(s)}\right)^{-\epsilon}\alpha\frac{\mathcal{E}^{i}(s)}{P_{NT}^{i}(s)}C_{T}^{i}(s)\\ &= \alpha\sum_{s}\pi(s|s_{-1})\left(p-\left(1+\tau_{L}^{i}\right)W^{i}(s)\right)\left(\frac{p}{P_{NT}^{i}(s)}\right)^{-\epsilon}\frac{1}{P_{NT}^{i}(s)}\end{aligned}$$

The first-order condition yields

$$P_{NT}^{ij}=\left(1+\tau_{L}^{i}\right)\frac{\epsilon}{\epsilon-1}\frac{\sum_{s}\pi(s|s_{-1})\left(\frac{1}{P_{NT}^{i}(s)}\right)^{1-\epsilon}W^{i}(s)}{\sum_{s}\pi(s|s_{-1})\left(\frac{1}{P_{NT}^{i}(s)}\right)^{1-\epsilon}} \tag{37}$$

Hence, all firms set the same price $P_{NT}^{i,j}=P_{NT}^{i}=P_{NT}^{i}(s)\ \forall s$, and (37) simplifies to

$$P_{NT}^{i}=\left(1+\tau_{L}^{i}\right)\frac{\epsilon}{\epsilon-1}\sum_{s}\pi(s|s_{-1})W^{i}(s) \tag{38}$$

From the household labor supply condition (6) we know that

$$\frac{W^{i}(s)}{P_{NT}^{i}}=C_{NT}^{i}(s)\left(N^{i}(s)\right)^{\phi}=1-\tau^{i}(s)$$

where the latter equality holds because price dispersion is $\triangle^i_{NT}(s) = 1$ in all states. We obtain

$$\sum_s \pi(s|s_{-1})\,\tau^i(s) = 1 - \frac{1-\frac{1}{\epsilon}}{1+\tau^i_L}$$

Since in each country the labor subsidy is set at the level $\tau^i_L = -\frac{1}{\epsilon}$, equation (20) obtains. Note that if the subsidy is set at any level $\tau^i_L > -\frac{1}{\epsilon}$, the optimal price-setting problem is to equalize the expected labor wedge to a strictly positive number, a condition inconsistent with the central bank's exchange-rate setting policy.

To understand the logic behind the equilibrium determination of relative prices, we further characterize the price level consistent with a given expectation of the central banks' exchange rate policy $\{\mathcal{E}^i(s)\}$. Using market clearing (11) with $\triangle^i_{NT}(s) = 1$, we obtain $\frac{W^i(s)}{P^i_{NT}} = \left(C^i_{NT}(s)\right)^{1+\phi}$. Using (4) and (9) into equation (38), we then find

$$(1+\tau_L)\frac{\epsilon}{\epsilon-1}\left(\sum_s \pi(s|s_{-1})\,C^i_{NT}(s)^{1+\phi}\right) = (1+\tau_L)\frac{\epsilon}{\epsilon-1}\left(\sum_s \pi(s|s_{-1})\left(\frac{\alpha\mathcal{E}^i(s)\,C^i_T(s)}{P^i_{NT}}\right)^{1+\phi}\right) = 1$$

and results in an equilibrium price level of

$$P^i_{NT} = \left((1+\tau_L)\frac{\epsilon}{\epsilon-1}\right)^{\frac{1}{1+\phi}} \alpha\left(\sum_{s\in\mathbf{S}} \pi(s|s_{-1})\left(\mathcal{E}^i(s)\,C^i_T(s)\right)^{1+\phi}\right)^{\frac{1}{1+\phi}} \tag{39}$$

When $\tau^i_L > -\frac{1}{\epsilon}$, monopolists collectively target a price that is higher than an average of $\mathcal{E}^i(s)\,C^i_T(s)$. This is inconsistent with central bank optimality. For example, under independent monetary policy, the central bank's exchange rate decision (33) leads to $\mathcal{E}^i(s)\,C^i_T(s) = \frac{1}{\alpha}P^i_{NT}$ in every state, and it is easy to see that these equations do not have a fixed point with positive prices. □

Proof of Lemma 6. Suppressing the dependence on s^{t-1}, the central bank's exchange rate choice (35) in the absence of price dispersion is

$$\mathcal{E}(s) = \frac{1}{\alpha}\left(\frac{1}{2}\left(\frac{P^1_{NT}}{C^1_T(s)}\right)^{-(1+\phi)} + \frac{1}{2}\left(\frac{P^2_{NT}}{C^2_T(s)}\right)^{-(1+\phi)}\right)^{-\frac{1}{1+\phi}} \tag{40}$$

while the monopolists' price setting conditions under the labor subsidy $\tau^1_L = \tau^2_L = -\frac{1}{\epsilon}$ lead to nontradable prices given by (39)

$$P^1_{NT} = \alpha\left(\sum_{s\in\mathbf{S}} \pi(s|s_{-1})\left(\mathcal{E}(s)\,C^1_T(s)\right)^{1+\phi}\right)^{\frac{1}{1+\phi}} \tag{41}$$

$$P^2_{NT} = \alpha\left(\sum_{s\in\mathbf{S}} \pi(s|s_{-1})\left(\mathcal{E}(s)\,C^2_T(s)\right)^{1+\phi}\right)^{\frac{1}{1+\phi}} \tag{42}$$

Define $\rho \equiv \frac{P_{NT}^2}{P_{NT}^1}$ as the relative price of nontradables in the two countries. Exploiting the homogeneity of equations, using (40) we obtain that

$$\frac{\mathcal{E}(s)}{P_{NT}^1} = \frac{1}{\alpha}\left(\frac{1}{2}\left(\frac{1}{C_T^1(s)}\right)^{-(1+\phi)} + \frac{1}{2}\left(\frac{\rho}{C_T^2(s)}\right)^{-(1+\phi)}\right)^{-\frac{1}{1+\phi}} \tag{43}$$

and this allows to rewrite (41) as

$$\begin{aligned} 1 &= \alpha\left(\sum_{s\in\mathbf{S}}\pi(s|s_{-1})\left(\frac{\mathcal{E}(s)}{P_{NT}^1}C_T^1(s)\right)^{1+\phi}\right)^{\frac{1}{1+\phi}} \\ &= \left(\sum_{s\in\mathbf{S}}\pi(s|s_{-1})\left(\frac{1}{2}+\frac{1}{2}\left(\frac{\rho C_T^1(s)}{C_T^2(s)}\right)^{-(1+\phi)}\right)^{-1}\right)^{\frac{1}{1+\phi}} \end{aligned}$$

The relative price ρ is therefore a solution to

$$\sum_{s\in\mathbf{S}}\pi(s|s_{-1})\left(\frac{1}{2}+\frac{1}{2}\left(\frac{C_T^2(s)}{\rho C_T^1(s)}\right)^{(1+\phi)}\right)^{-1} = 1 \tag{44}$$

The function $\rho \to \sum_{s\in\mathbf{S}}\pi(s|s_{-1})\left(\frac{1}{2}+\frac{1}{2}\left(\frac{C_T^2(s)}{\rho C_T^1(s)}\right)^{(1+\phi)}\right)^{-1}$ is strictly increasing in ρ, with limits $\frac{1}{2}$ as $\rho \to 0$ and 2 as $\rho \to \infty$, so (44) has a unique solution. Equation (44) then delivers a unique solution for all relative prices $\frac{\mathcal{E}(s)}{P_{NT}^1}$, $s \in \mathbf{S}$, and therefore for $\frac{\mathcal{E}(s)}{P_{NT}^2} = \frac{\mathcal{E}(s)}{P_{NT}^1}\frac{1}{\rho}$, $s \in \mathbf{S}$ also. These relative prices are consistent with (42) because satisfaction of (44) ensures that

$$\begin{aligned} 0 &= \sum_{s\in\mathbf{S}}\pi(s|s_{-1})\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{C_T^2(s)}{\rho C_T^1(s)}\right)^{(1+\phi)}}{\frac{1}{2}+\frac{1}{2}\left(\frac{C_T^2(s)}{\rho C_T^1(s)}\right)^{(1+\phi)}} \\ &= \sum_{s\in\mathbf{S}}\pi(s|s_{-1})\frac{\frac{1}{2}\left(\frac{\rho C_T^1(s)}{C_T^2(s)}\right)^{(1+\phi)}-\frac{1}{2}}{\frac{1}{2}\left(\frac{\rho C_T^1(s)}{C_T^2(s)}\right)^{(1+\phi)}+\frac{1}{2}} \\ &= \sum_{s\in\mathbf{S}}\pi(s|s_{-1})\left(1-\frac{1}{\frac{1}{2}\left(\frac{\rho C_T^1(s)}{C_T^2(s)}\right)^{(1+\phi)}+\frac{1}{2}}\right) \end{aligned}$$

so we also have

$$\sum_{s\in\mathbf{S}}\pi(s|s_{-1})\left(\frac{1}{2}+\frac{1}{2}\left(\frac{\rho C_T^1(s)}{C_T^2(s)}\right)^{(1+\phi)}\right)^{-1} = 1$$

This result is another illustration of the redundancy of equations: using $\tau^i(s)$ to denote the labor wedge in country i and state s, price-setting in country 1 implies $\sum\pi(s|s_{-1})\tau^1(s) = 0$, and monetary policy implies $\tau^1(s) + \tau^2(s) = 0$ for all s, from we obtain $\sum \pi(s|s_{-1})\,\tau^2(s) = 0$. □

Another proof of theorem 1. Following the proof of Lemma 6, consider the special case where transfers ensure full risk-sharing ($\frac{C_T^2(s)}{C_T^1(s)} = \lambda$ for all states s). The unique solution to (44) is then clearly $\rho = \lambda$. In this case $\frac{\mathcal{E}(s)}{P_{NT}^1} = \frac{1}{\alpha C_T^1(s)}, \frac{\mathcal{E}(s)}{P_{NT}^2} = \frac{1}{\alpha C_T^2(s)}$, and nontradables are at their efficient level in both countries and in all states. This is the risk-sharing miracle. □

Sufficient conditions for assumption 3. Consider the period indirect utility function attained by country i when choosing to make transfer T, taking as given the transfer T^{-i} made by the other country as well as the equilibrium reaction of the central bank to $\{T, T^{-i}\}$, which we denote $\mathcal{E}(T)$ to make the dependence of the exchange rate on the transfer explicit. Denote $C_T^i(T) = E_T^i + T^{-i} - T$ and $C_{NT}^i(T) = \alpha\frac{\mathcal{E}(T)}{P_{NT}^i}C_T^i(T)$. Note that

$$
\begin{aligned}
\frac{dC_{NT}^i(T)}{dT} &= \alpha\frac{1}{P_{NT}^i}\left(\mathcal{E}'(T)\,C_T^i(T) - \mathcal{E}(T)\right) \\
&= -\alpha\frac{\mathcal{E}(T)}{P_{NT}^i}\left(1 - \frac{\mathcal{E}'(T)}{\mathcal{E}(T)}C_T^i(T)\right) \\
&= -\frac{C_{NT}^i(T)}{C_T^i(T)}\left(1 - \frac{\mathcal{E}'(T)}{\mathcal{E}(T)}C_T^i(T)\right)
\end{aligned}
$$

In a Nash equilibrium where current fiscal policy only has impact on current utility, T is chosen to maximize

$$
v^i(T) = \log\left(C_T^i(T)\right) + \alpha\left(\log\left(C_{NT}^i(T)\right) - \frac{\left(C_{NT}^i(T)\right)^{1+\phi}}{1+\phi}\right)
$$

The first-order condition of this problem is

$$
\begin{aligned}
\frac{dv^i(T)}{dT} &= -\frac{1}{C_T^i(T)} - \alpha\frac{C_{NT}^i(T)}{C_T^i(T)}\left(1 - \frac{\mathcal{E}'(T)}{\mathcal{E}(T)}C_T^i(T)\right)\left(\frac{1}{C_{NT}^i(T)} - C_{NT}^i(T)^{\phi}\right) \\
&= -\frac{1}{C_T^i(T)}\left[1 + \alpha\left(1 - \frac{\mathcal{E}'(T)}{\mathcal{E}(T)}C_T^i(T)\right)\underbrace{\left(1 - C_{NT}^i(T)^{1+\phi}\right)}_{\tau^i(T)}\right]
\end{aligned}
$$

where $\tau^i(T)$ is country i's labor wedge, a determined by the central bank's reaction to (T, T^{-i}) and to the prices in place. From (34), this is done to enforce

$$
\frac{1}{2}\left(\alpha\frac{\mathcal{E}(T)}{P_{NT}^1}C_T^1(T)\right)^{1+\phi} + \frac{1}{2}\left(\alpha\frac{\mathcal{E}(T)}{P_{NT}^2}C_T^2(T)\right)^{1+\phi} = 1
$$

differentiating we find

$$\frac{1}{2}\left(\alpha\frac{\mathcal{E}(T)}{P_{NT}^{1}}C_{T}^{1}(T)\right)^{1+\phi}\left[\frac{\mathcal{E}'(T)}{\mathcal{E}(T)}+\frac{C_{T}^{1\prime}(T)}{C_{T}^{1}(T)}\right]+\frac{1}{2}\left(\alpha\frac{\mathcal{E}(T)}{P_{NT}^{2}}C_{T}^{2}(T)\right)^{1+\phi}\left[\frac{\mathcal{E}'(T)}{\mathcal{E}(T)}+\frac{C_{T}^{2\prime}(T)}{C_{T}^{2}(T)}\right]=0$$

which leads to the central bank adjustment rule

$$\frac{\mathcal{E}'(T)}{\mathcal{E}(T)}=-\frac{1}{2}\left(1-\tau^{1}(T)\right)\frac{C_{T}^{1\prime}(T)}{C_{T}^{1}(T)}-\frac{1}{2}\left(1-\tau^{2}(T)\right)\frac{C_{T}^{2\prime}(T)}{C_{T}^{2}(T)}$$

For country i, an increase in own transfer reduces home country tradable consumption and increases foreign: $C_{T}^{i\prime}(T)=-1$ and $C_{T}^{-i\prime}(T)=1$, so

$$\begin{aligned}\frac{\mathcal{E}'(T)\,C_{T}^{i}(T)}{\mathcal{E}(T)} &= \frac{1}{2}\left(1-\tau^{i}(T)\right)-\frac{1}{2}\left(1-\tau^{-i}(T)\right)\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}\\ &= \frac{1}{2}\left(1-\tau^{i}(T)\right)-\frac{1}{2}\left(1+\tau^{i}(T)\right)\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}\\ &= \frac{1}{2}\left(1-\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}\right)-\frac{1}{2}\tau^{i}(T)\left(1+\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}\right)\end{aligned}$$

and finally

$$1-\frac{\mathcal{E}'(T)\,C_{T}^{i}(T)}{\mathcal{E}(T)}=\frac{1}{2}\left(1+\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}\right)\left(1+\tau^{i}(T)\right)$$

an expression which is always strictly positive, given that $\tau^{i}(T)>-1$. We therefore have

$$\frac{dv^{i}(T)}{dT}=-\frac{1}{C_{T}^{i}(T)}\left[1+\alpha\frac{1}{2}\left(1+\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}\right)\left(1+\tau^{i}(T)\right)\tau^{i}(T)\right]$$

The function $\tau\mapsto 1+\alpha\frac{1}{2}\left(1+\frac{C_{T}^{i}}{C_{T}^{-i}}\right)(1+\tau)\,\tau$ has value 1 at both $\tau=0$ and $\tau=-1$. Its minimum $1-\alpha\frac{1}{8}\left(1+\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}\right)$ is attained at $\tau=-\frac{1}{2}$. If

$$\alpha<\frac{8}{1+\frac{C_{T}^{i}(T)}{C_{T}^{-i}(T)}}$$

then $\frac{v^{i}(T)}{dT}<0$ and country i prefers not to make a transfer.

A sufficient condition for countries not to want to make transfers in autarky is then

$$\alpha<\frac{8}{1+\max_{s}\left\{\frac{E_{T}^{1}(s)}{E_{T}^{2}(s)};\frac{E_{T}^{2}(s)}{E_{T}^{1}(s)}\right\}}$$

more generally, the condition is that $1+\alpha\frac{1}{2}\left(1+\frac{E_T^i(s)}{E_T^j(s)}\right)\left(1+\tau^i\left(s\right)\right)\tau^i\left(s\right)>0$ for the labor wedge in country i and state s that results from central bank optimization given an autarkic fiscal policy, that is

$$\tau^i\left(s\right)=1-\frac{1}{\frac{1}{2}+\frac{1}{2}\left(\frac{\rho E_T^i(s)}{E_T^j(s)}\right)^{-(1+\phi)}}$$

where ρ is the solution to (44). □

Proof of Lemma 8. Suppose that countries have endowment processes governed by assumption 5 and follow Markov transfer rules. From (44), given any z_{-1}, the relative nontradables price $\rho=\frac{P_{NT}^2(z_{-1})}{P_{NT}^1(z_{-1})}$ solves

$$\sum_{s\in\mathbf{S}}\pi\left(z|z_{-1}\right)\left\{\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\left(\frac{E^H\left(z\right)-T\left(z,z_{-1}\right)}{\rho\left(E^L\left(z\right)+T\left(z,z_{-1}\right)\right)}\right)^{(1+\phi)}\right)^{-1}+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\left(\frac{E^L\left(z\right)+T\left(z,z_{-1}\right)}{\rho\left(E^H\left(z\right)-T\left(z,z_{-1}\right)\right)}\right)^{(1+\phi)}\right)^{-1}\right\}=1 \tag{45}$$

making use of the equality

$$\frac{1}{2}\frac{1}{\frac{1}{2}+\frac{1}{2}x}+\frac{1}{2}\frac{1}{\frac{1}{2}+\frac{1}{2}\frac{1}{x}}=1\quad\forall x$$

applied separately to each pair $x=\left(\frac{E^H(z)-T(z,z_{-1})}{E^L(z)+T(z,z_{-1})}\right)^{(1+\phi)}$, we see that $\rho=1$ is a solution to (45), hence its unique solution by Lemma 6. □

Proof of theorem 3. By Lemma 8, the relative nontradables price in monetary union is $\rho=1$. This delivers the indirect utility function

$$\tilde{v}\left(C\right)=\log\left(C\right)+\alpha\left(\log\left(\alpha\epsilon_z\left(C\right)C\right)-\frac{\left(\alpha\epsilon_z\left(C\right)C\right)^{1+\phi}}{1+\phi}\right)$$

where the common real exchange rate for both countries is given by

$$\epsilon_z\left(C\right)=\frac{1}{\alpha}\left(\frac{1}{2}\left(\frac{1}{C}\right)^{-(1+\phi)}+\frac{1}{2}\left(\frac{1}{E\left(z\right)-C}\right)^{-(1+\phi)}\right)^{-\frac{1}{1+\phi}} \tag{46}$$

with $E\left(z\right)=E^H\left(z\right)+E^L\left(z\right)$. This implies that

$$g_z\left(C\right)\equiv\alpha\epsilon_z\left(C\right)C=\left(\frac{1}{2}+\frac{1}{2}\left(\frac{C}{E\left(z\right)-C}\right)^{-(1+\phi)}\right)^{-\frac{1}{1+\phi}} \tag{47}$$

is a strictly increasing function of C. $g_z\left(C\right)$ attains 1 at $C=\frac{E(z)}{2}$.

The proof of theorem (3) now follows. By definition of subgame perfection, the H country does not want to refrain from the transfer at any node z^t:

$$\log\left(C_T^H\left(z^t\right)\right)+\beta\sum_{z^h\succeq z^t}\frac{\pi\left(z^h|z^t\right)}{2}\left(\log\left(C_T^H\left(z^t\right)\right)+\log\left(C_T^L\left(z^t\right)\right)\right)$$
$$\geq\log\left(E_T^H\left(z^t\right)\right)+\beta\sum_{z^h\succeq z^t}\frac{\pi\left(z^h|z^t\right)}{2}\left(\log\left(E_T^H\left(z^t\right)\right)+\log\left(E_T^L\left(z^t\right)\right)\right)$$

Using the Markov structure, the participation constraints for all $z\in\mathbf{Z}$ can be written

$$\underbrace{\log\left(E_T^H(z)\right)-\log\left(C_T^H(z)\right)}_{\text{One-shot gain from defaulting}}\leq\underbrace{\beta\sum_{z}\frac{\tilde{\pi}\left(z'|z\right)}{2}\left[\left(\log\left(C_T^L\left(z'\right)\right)-\log\left(E^L\left(z'\right)\right)\right)-\left(\log\left(E^H\left(z'\right)\right)-\log\left(C_T^H\left(z'\right)\right)\right)\right]}_{\text{Expected loss from lack of future risk-sharing}}$$

where $\tilde{\pi}\left(z'|z\right)$ are the elements of the matrix $\tilde{\Pi}=\Pi^z\left(I-\beta\Pi^z\right)^{-1}$, which take into account the relevant mix of future probabilities and discounting. Due to the ordering (25) and the concavity of log, we have

$$\left(\log\left(C_T^L(z)\right)-\log\left(E^L(z)\right)\right)-\left(\log\left(E^H(z)\right)-\log\left(C_T^H(z)\right)\right)\geq 0$$

Consider sustaining the same SPE under monetary union using the same on-path and off-path actions. Lemma 8 implies that both countries have the same real exchange rate at every node, so they evaluate tradable consumption levels using the same indirect utility function

$$\tilde{v}_z(C)=\log(C)+\alpha\left(\log\left(\alpha\epsilon_z(C)\,C\right)-\frac{1}{1+\phi}\left(\alpha\epsilon_z(C)\,C\right)^{1+\phi}\right)$$

where $\epsilon_z(C)$ is given in (46). Recall from (47) that $g_z(C)\equiv\alpha\epsilon_z(C)\,C$ is strictly monotone and attains the value 1 (the efficient nontradables consumption level) at $C=\frac{E^H(z)+E^L(z)}{2}$. This is a consequence of the risk-sharing miracle. Since

$$f(x)\equiv\alpha\left(\log(x)-\frac{1}{1+\phi}x^{1+\phi}\right)$$

is a concave function with a maximum at $x=1$, the function $\tilde{v}_z(C)-\log(C)=f\left(g_z(C)\right)$ is single-peaked with a maximum at $\frac{E^L(z)+E^H(z)}{2}$. In particular

$$\begin{aligned} f\left(g_z\left(E^H(z)\right)\right) &\leq f\left(g_z\left(C_T^H(z)\right)\right) \quad \forall z\\ f\left(g_z\left(E^L(z)\right)\right) &\leq f\left(g_z\left(C_T^L(z)\right)\right) \quad \forall z\end{aligned}$$

from which it follows that

$$\tilde{v}_z\left(E^H(z)\right)-\tilde{v}_z\left(C_T^H(z)\right)\leq\log\left(E^H(z)\right)-\log\left(C_T^H(z)\right)\quad\forall z$$

and that

$$\log\left(C_T^L(z)\right) - \log\left(E^L(z)\right) \le \tilde{v}_z\left(C_T^L(z)\right) - \tilde{v}_z\left(E^L(z)\right) \quad \forall z$$

Combining these inequalities, we obtain

$$\begin{aligned}
\tilde{v}_z\left(E_T^H(z)\right) - \tilde{v}_z\left(C_T^H(z)\right) &\le \log\left(E_T^H(z)\right) - \log\left(C_T^H(z)\right) \\
&\le \beta\sum_z \frac{\tilde{\pi}(z'|z)}{2}\left[\left(\log\left(C_T^L(z)\right) - \log\left(E^L(z)\right)\right) - \left(\log\left(E^H(z)\right) - \log\left(C_T^H(z)\right)\right)\right] \\
&\le \beta\sum_z \frac{\tilde{\pi}(z'|z)}{2}\left[\left(\tilde{v}_z\left(C_T^L(z)\right) - \tilde{v}_z\left(E^L(z)\right)\right) - \left(\tilde{v}_z\left(E^H(z)\right) - \tilde{v}_z\left(C_T^H(z)\right)\right)\right]
\end{aligned}$$

which guarantees that the participation constraint for the H country is met in every state z under monetary union, as claimed. □

Proof of theorem 4. We consider the simplest possible example of our framework: the symmetric iid case with union-wide tradable output equal to 1. In the notation above, the Markov chain is reduced to a point $z = 1$ in every period, $E^H = e$ and $E^L = 1 - e$, with $e > \frac{1}{2}$. In this context, a Markov transfer is a value T, such that $\left(C_T^L, C_T^H\right) = (1 - e + T, e - T)$. We consider transfers that improve risk-sharing, in other words $T \in \left[0, \frac{1}{2} - e\right]$.

Under independent monetary policy, given the flow value from the nontradables side is always constant at $f^* = f(1) \equiv -\frac{\alpha}{1+\phi}$, the value of being in the high state under the contract is

$$V^H(T) = \log(e - T) + \frac{\beta}{1-\beta}\left(\frac{1}{2}\log(e-T) + \frac{1}{2}\log(1-e+T)\right) + \frac{f^*}{1-\beta}$$

The participation constraint states that $V^H(T) \ge V^H(0)$. Since V^H is concave in T, there exists a $T > 0$ such that this constraint is satisfied if, and only if

$$\left.\frac{dV^H}{dT}\right\rfloor_{T=0} = -\frac{1}{e} + \frac{\beta}{1-\beta}\frac{1}{2}\left(-\frac{1}{e} + \frac{1}{1-e}\right) \ge 0$$

that is, if and only if

$$\beta \ge \underline{\beta}^{indep} = 2(1-e)$$

Suppose countries now are sharing risks perfectly $T = \frac{1}{2} - e$, therefore obtaining $\left(C_T^L, C_T^H\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$ and value

$$V^{FB} = \frac{1}{1-\beta}\left(\log\left(\frac{1}{2}\right) + f^*\right)$$

Consider the deviation of the country in the high state, which is punished by the grim trigger strategy. This deviation is valuable if

$$\underbrace{\log(e) - \log\left(\frac{1}{2}\right)}_{\text{One-shot gain from defaulting}} \ge \frac{\beta}{1-\beta}\frac{1}{2}\underbrace{\left(2\log\frac{1}{2} - \log(e) - \log(1-e)\right)}_{\text{Expected loss from lack of future risk-sharing}}$$

Which is equivalent to the condition

$$\beta \leq \overline{\beta}^{indep} = 2\frac{\log(e) - \log\left(\frac{1}{2}\right)}{\log(e) - \log(1-e)} \leq 1$$

Consider now the case of a monetary union, and assume countries are achieving perfect risk-sharing $T = \frac{1}{2} - e$. Due to the risk-sharing miracle (theorem 1), their nontradables side is perfectly stabilized and they obtain V^{FB}. However, a deviation now entails the additional cost

$$c(e) = f^* - f(g(e)) = f(1) - f(g(e)) = \alpha\left(\frac{g(e)^{1+\phi} - 1}{1+\phi} - \log(g(e))\right) \geq 0$$

where nontradable consumption, considering the central bank reaction, is

$$g(e) = \left(\frac{1}{2} + \frac{1}{2}\left(\frac{e}{1-e}\right)^{-(1+\phi)}\right)^{-\frac{1}{1+\phi}} > 1$$

for the country in the H state and and $g(1-e) < 1$ for the country in the L state. The condition for a profitable deviation is now

$$\log(e) - \log\left(\frac{1}{2}\right) - c(e) \geq \frac{\beta}{1-\beta}\frac{1}{2}\left(2\log\frac{1}{2} - \log(e) - \log(1-e) + c(e) + c(1-e)\right)$$

which shows clearly that monetary union both lowers the benefit and raises the costs of defaulting. A country therefore finds it profitable to deviate from first-best risk-sharing under monetary union when

$$\beta \leq \overline{\beta}^{union}(\alpha, \phi, e) = 2\frac{\log(e) - \log\left(\frac{1}{2}\right) - c(e)}{\log(e) - \log(1-e) + c(1-e) - c(e)}$$

It is simple to show that $\overline{\beta}^{union}(\alpha, \phi, e)$ is strictly decreasing in α with $\lim_{\alpha\to\infty} \overline{\beta}^{union}(\alpha, \phi, e) < 0$. This has two consequences. First,

$$\overline{\beta}^{union}(\alpha, \phi, e) \leq \overline{\beta}^{union}(0, \phi, e) = \overline{\beta}^{indep}$$

formalizing our claim that first-best risk-sharing is easier to sustain under monetary union than under independent monetary policy. Second, there always exists values of (α, ϕ, e) such that $\overline{\beta}^{union}(\alpha, \phi, e) \leq \underline{\beta}^{indep}$. To complete the claim that countries with discount factors $\overline{\beta}^{union}(\alpha, \phi, e) \leq \beta \leq \underline{\beta}^{indep}$ can only sustain autarky under independent monetary policy, but can sustain full risk-sharing under monetary union, one also needs to check that autarky under monetary union is indeed subgame perfect at those parameters. This is ensured by the condition

$$\frac{1}{e} + \frac{\alpha}{g(e)}\left(1 - (g(e))^{1+\phi}\right)g'(e) \geq 0$$

or equivalently

$$\alpha \leq \overline{\alpha}\left(e,\phi\right) = \frac{1}{\frac{\left(\frac{e}{1-e}\right)^{-(1+\phi)}}{1+\left(\frac{e}{1-e}\right)^{-(1+\phi)}}\frac{(g(e))^{1+\phi}-1}{1-e}}$$

□